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## ARNOLD WILSON

*Registered Professional Engineer*

R.F.D. Box 208  
SPRINGVILLE, UTAH

March 19, 1979

Harold Jackson  
Storage Structures Engineer  
Structures & Environmental Section  
Research Branch, Agriculture Canada, Bld. 74  
Ottawa, Ontario, Canada K1A0C6

Dear Harold:

Results of a finite element computer analysis for the two inch thick concrete domes at Lethbridge, Alberta, Canada are included in this report.

Maximum stresses and moments caused by the unsymmetrical loads which you specified, with a maximum unit value of 62 pounds per square foot, are summarized in the following table:

	Max. Radial Stress (psi)	Max. Hoop Stress (psi)	Bending Moment
Pinned Base	83 comp. 29 tens.	84 comp. 36 tens.	10.0 lb-in/inch
Fixed Base	87 comp. 30 tens.	80 comp. 30 tens.	9.9 lb-in/inch
Dome-Ring Dead Load	47 comp. no tens	43 comp. tens. at base	only at base

The maximum tension force in the ring at the base of the dome, as calculated from the pinned base analysis, was 20 kips. The maximum tension in the ring from the dead load of the dome structure was 19 kips resulting in a maximum ring force of 39 kips compared to a maximum ring force of 40 kips due to the dead load plus a snow load of 40 pounds per square foot.

The bending moment causes compression in the shell which is compared as follows:

From working stress design method

$$M = \frac{F_c}{2} (b)(kd) \left(d - \frac{kd}{2}\right) = Rbd^2$$

Therefore

$$R = \frac{M}{bd^2} = \frac{10.0 \text{ lb-in}}{(1)(1)^2} = 10 \text{ psi} < 226 \text{ psi} \quad \text{o.k.}$$

The maximum value of R, for a concrete strength of  $f'_c = 3 \text{ ksi}$  and for a steel strength of  $f_y = 40 \text{ ksi}$ , is 226 psi. Reference; "Working Stress Design Handbook", American Concrete Institute, SP-3

The maximum stresses summarized in the previous table are at different locations for the dead load and the unsymmetrical loading, however, even if these values were superimposed together the result would be less than one-tenth of the maximum allowable stress of  $0.45 f'_c$ . As indicated from the bending analysis the resulting stresses are less than one-twentieth of the allowable value. Bending stresses do not occur under uniformly distributed loads such as dead load or snow load except adjacent to the edge beam at the bottom of the dome. The bending caused in the upper portions of the dome as a result of the unsymmetrical loading resulted in low stresses not critical to the safety of the structure.

Arches resist unsymmetrical loads by radial or meridional forces and bending. Therefore, when unsymmetrical loads are applied to arches, serious bending is included in the analysis and design. Domes, because of the double curvature, resist unsymmetrical loads with radial or meridional forces and in addition with forces that are at right angles to the radial forces that are called hoop forces. The hoop forces, by increasing the resistance to unsymmetrical loading, restrain local bending such that it is usually not a critical design factor. Physically as an arch segment of the dome tends to bend under the unsymmetrical loading the hoop forces restrain it just as if stiff rings were wrapped around the structure.

In addition when edge forces are acting on a dome they create bending moments in a very narrow region near the edge and have generally no effect throughout a large portion of the structure, whereas edge forces in equilibrium applied to an arch propagate through the entire structural system and create large bending moments. A good discussion of Arch versus Dome can be found in "Thin Shell Concrete Structures", by Billington, McGraw Hill Book Co., 1965, P.2.

Even with this most unusual and unsymmetrical loading the stresses in the two-inch thick dome are well within safe and allowable limits.

This analysis shows that a dome is suited to almost any type of loading and that local bending induced by unsymmetrical loading is not critical as it is an arch.

Attached is a brief description of the SAP IV computer program that was used for the analysis of the unsymmetrical loading. This program has been widely distributed and used for the analysis of such complex structural systems.

Mr. Rauli Uitto was responsible for using the SAP IV computer program to analyze the dome structure because of his training and experience with this program. A brief resume of Mr. Uitto is attached for your information.

A copy of the computer output is also included for your information. Summary sheets showing the magnitude and location of the radial and hoop stresses for both a fixed base and a pinned base dome, 130 ft. in diameter, 40 ft. high and 2 inches thick are also included. Note that the results are similar in the upper part of the dome which is reasonable since the edge effects from the dome base dampen rapidly and have little influence over the major portion of the shell.

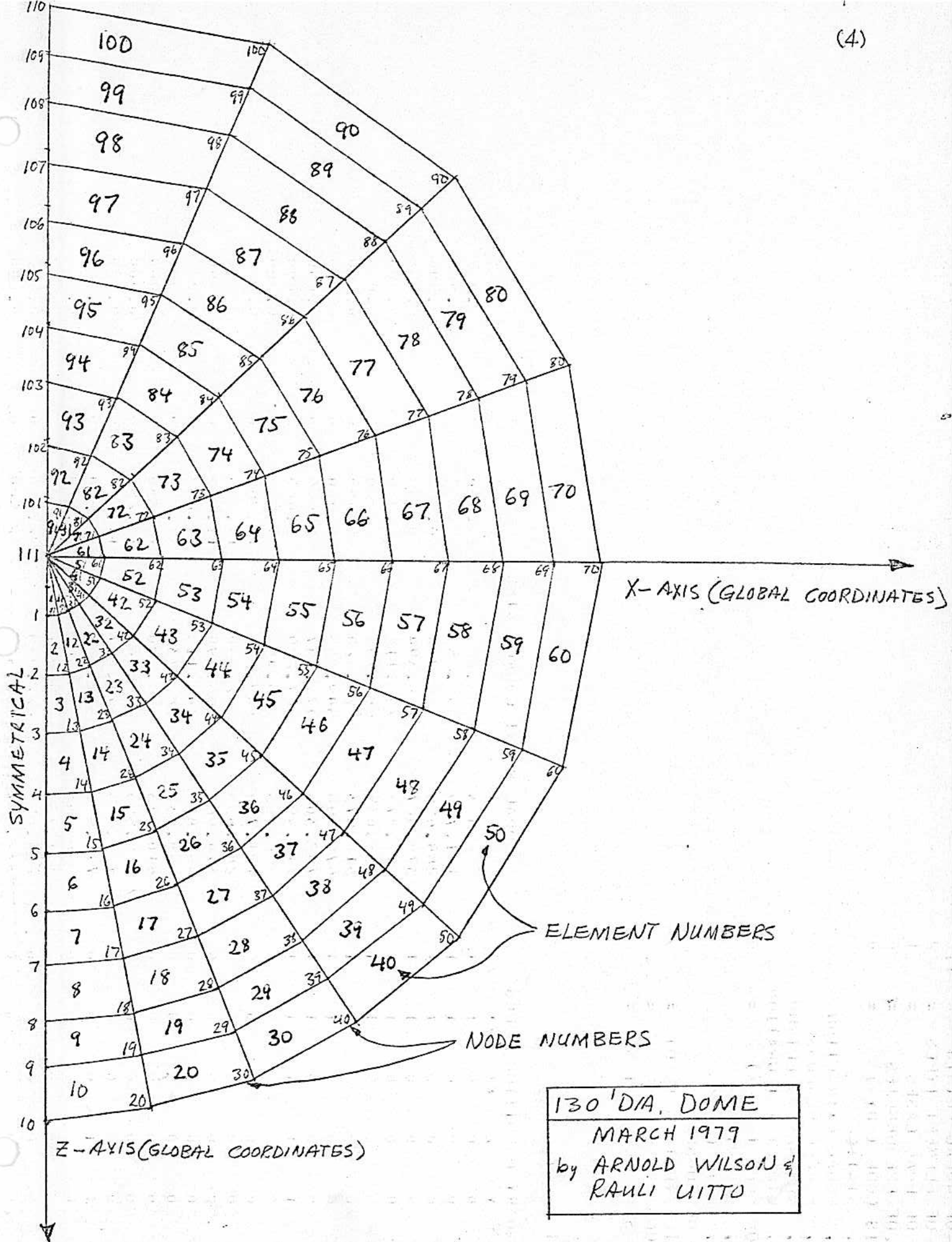
If I can answer any questions or be of further assistance please let me know.

Sincerely,

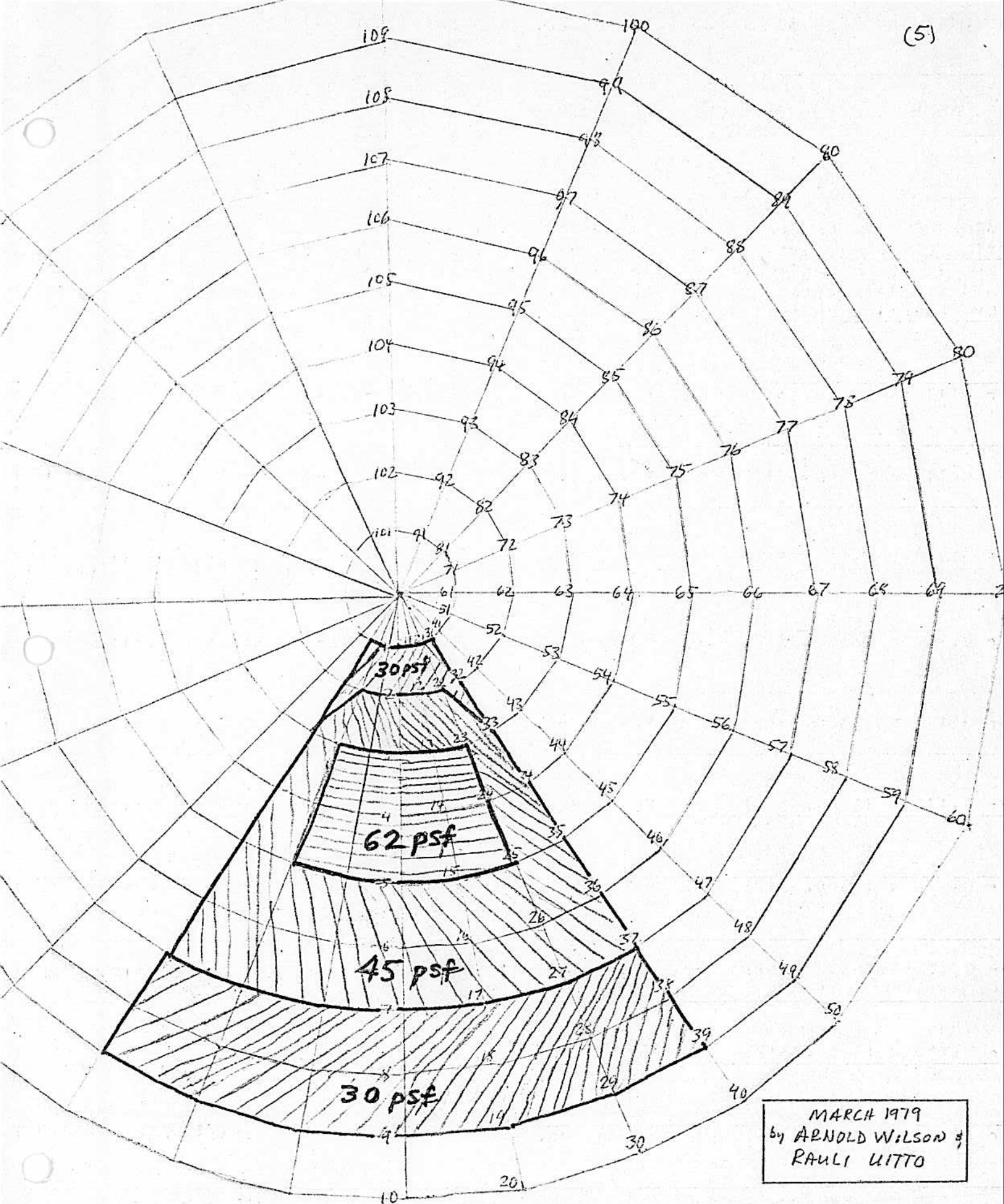


Arnold Wilson  
Structural Engineer

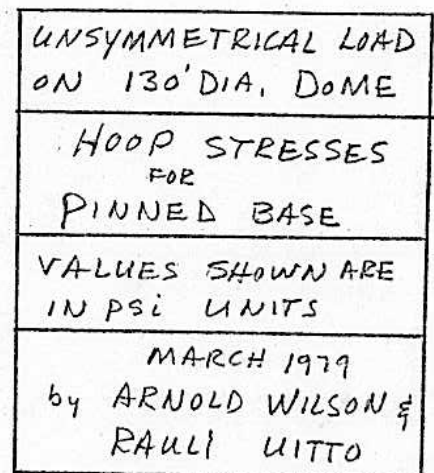
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c. Dennis Darby  
Ray Durksen  
David South

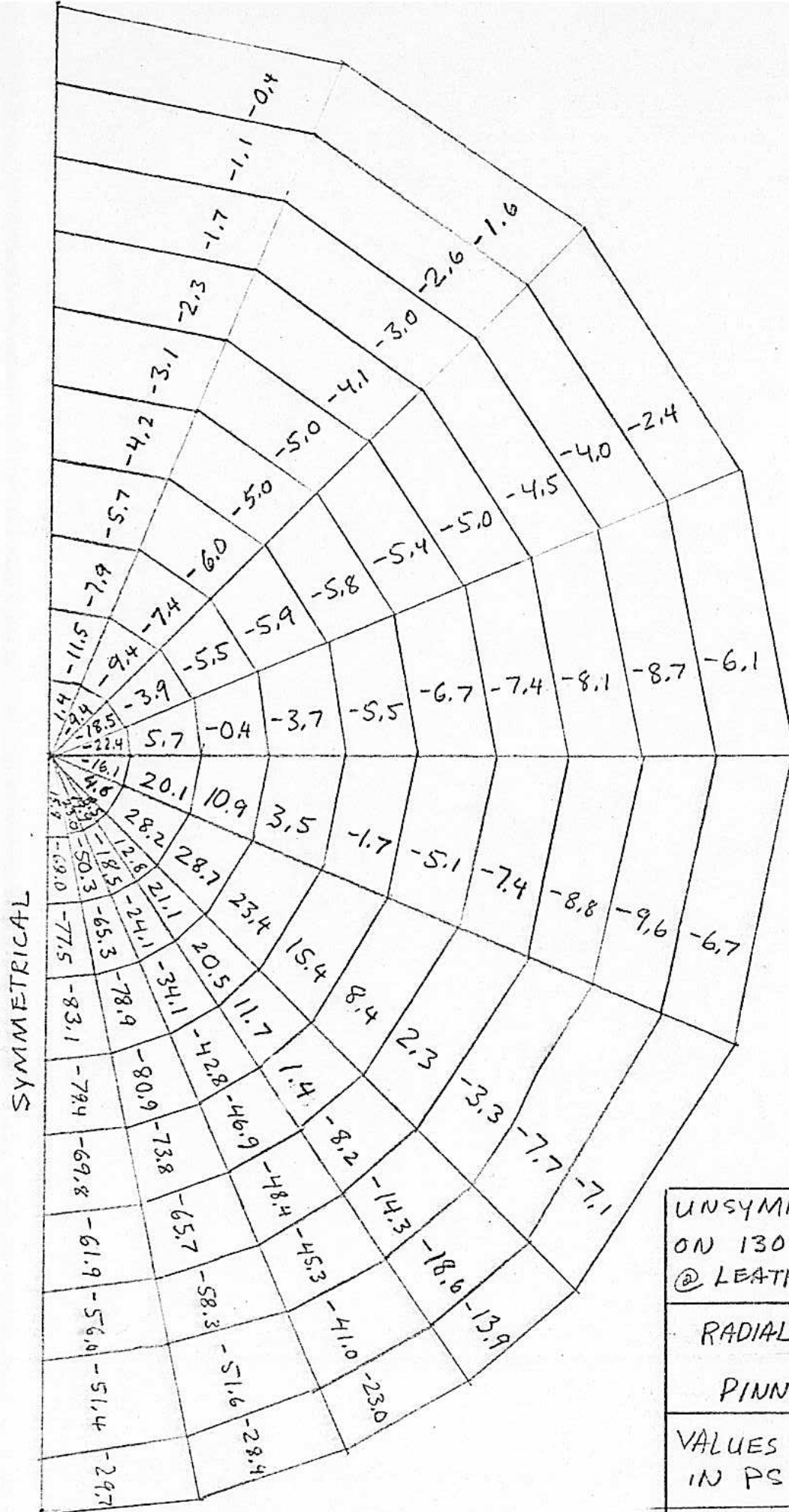






LOADING FOR 130' DIA. DOME  
DISTRIBUTED LOADING TAKEN AS POINT LOAD AT NODES





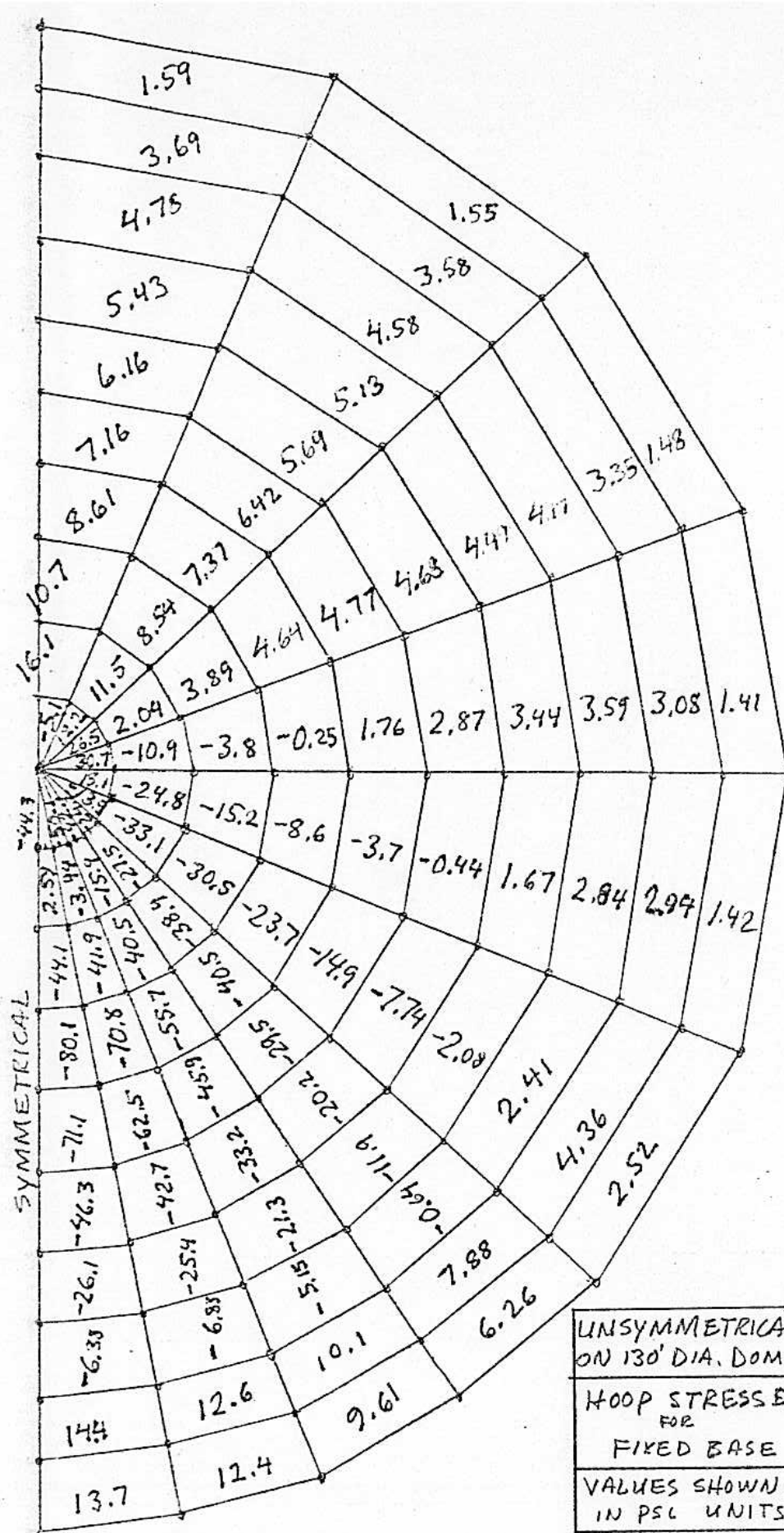
UNSYMMETRICAL LOAD  
ON 130' DIA. DOME  
@ LEATHBRIDGE, ALBERTA

# RADIAL STRESSES FOR PINNED BASE

VALUES SHOWN ARE  
IN PSI UNITS

MARCH 1979  
by ARNOLD WILSON &  
RAULI LITTO



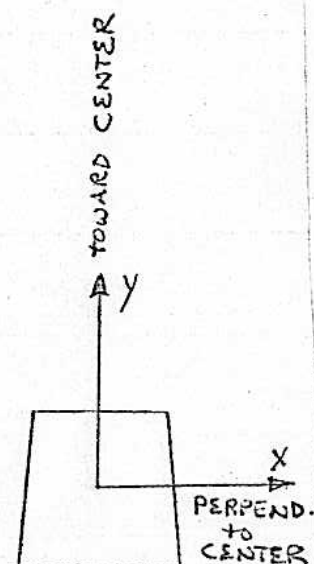


UNSYMMETRICAL LOAD  
ON 130' DIA. DOME

HOOP STRESSES  
FOR  
FIXED BASE

VALUES SHOWN ARE  
IN PSI UNITS

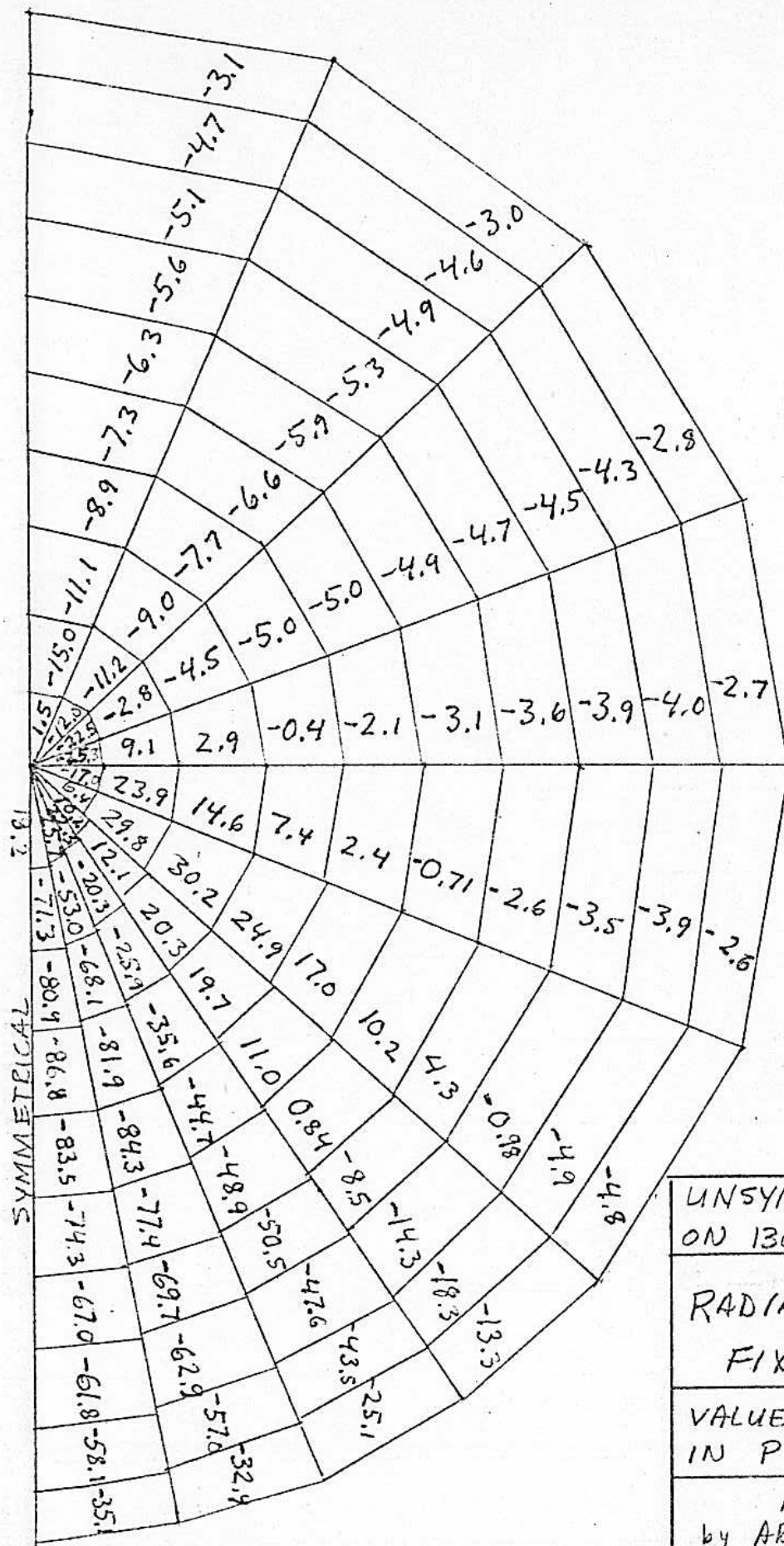
MARCH 1979  
by ARNOLD WILSON &  
RAULI UITTO



LOCAL COORDINATES

X - DIRECTION  
STRESSES (HOOP STRESS)





UNSYMMETRICAL LOAD  
ON 130' DIA. DOME

RADIAL STRESSES  
FOR  
FIXED BASE

VALUES SHOWN ARE  
IN PSI UNITS

MARCH 1979  
by ARNOLD WILSON &  
RAULI LITTO

Rauli Juhani Uitto

ADDRESS

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Provo, Utah 84602  
(801) 375-8078  
(801) 374-1211 ex. 2685

PERSONAL DATA

Birthdate: August 12, 1945  
Ht: 6' 0" Wt: 190 lbs  
Married - one child  
S.S. 529-76-9984

EDUCATION

- 1979 PhD. degree in Civil Engineering, Brigham Young University, Provo, Ut 84602  
1975 M.E. degree in Civil Engineering, Brigham Young University

Dissertation work in "Static and Dynamic Analysis of Cable-Stayed bridges using Finite Element Technique". Master's research in "Tensile Testing with Explosive Strain Rates". Course work includes thirty semester hours in graduate level structural analysis and design, twelve in materials, twenty-eight in thesis, dissertation and research, and thirty-three semester hours in mathematics and computer science. GPA 3.41 on a maximum 4.00 scale.

- 1972 B.S. degree in Physics, Brigham Young University  
B.S. equivalent in Civil Engineering (all requisite undergraduate courses)  
Forty-five semester units in physics, twenty-four in mathematics and computer science. Fifteen semester hours in undergraduate civil engineering.

WORK EXPERIENCE

- Jan 1979 to present Megadiamond Co. and Dr. Arnold Wilson  
Consulting assignments with finite element analysis of a diamond press cylinder and thin shell dome structures.  
1974 - 1979 BYU Civil Engineering Department  
Development of a technique to produce portland cement using solar energy. Student teaching several laboratory courses, grading for a variety of courses.  
Summer of 1976 Lockheed Electronics Co., Air Force Rocket Propulsion Laboratory (AFRPL), Edwards AFB, Calif. 93523.  
Scientific Programmer. Responsible for programming scientific and engineering problems with a CDC-6400 computer.  
1970 - 1973 BYU Physics Department  
Full time programmer for a fusion research program in summer and fall of 1972. Grader for several undergraduate physics classes.

EXTRACURRICULAR ACTIVITIES

Member of Sigma Xi and Society for Experimental Stress Analysis. Member of BYU track team 1967 - 1970 (javelin thrower - 253 feet), newspaper sports correspondent to Finland, chess player (USCF rating 1957).

REFERENCES: Dr. D. Allan Firmage/major advisor/structures/ BYU Civil Engineering Department (801) 374-1211 ex. 2811  
Thomas W. Sederberg/structures// Arnold Wilson/thin shell structures/BYU  
Dr. Wendell Horning/scientific consultant/ RPL, Edwards AFB, Ca 93523

*taught CE 508 Lab (2 semesters), CE 305 Lab, and 303 question and*

REPORT NO.  
EERC 73-11  
JUNE 1973  
REVISED APRIL 1974

EARTHQUAKE ENGINEERING RESEARCH CENTER

# SAP IV

A STRUCTURAL ANALYSIS PROGRAM  
FOR STATIC AND DYNAMIC RESPONSE  
OF LINEAR SYSTEMS

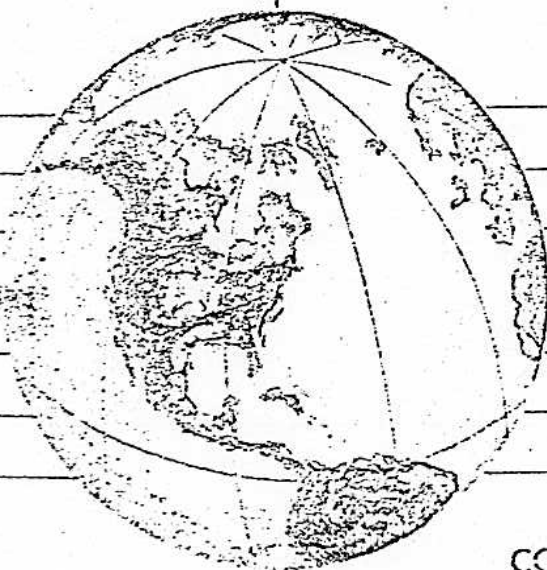
by

KLAUS-JÜRGEN BATHE

EDWARD L. WILSON

FRED E. PETERSON

A Report to the  
National Science Foundation



COLLEGE OF ENGINEERING

UNIVERSITY OF CALIFORNIA • Berkeley, California

## 1. INTRODUCTION

The development of an effective computer program for structural analysis requires a knowledge of three scientific disciplines -- structural mechanics, numerical analysis and computer application. The development of accurate and efficient structural elements requires a modern background in structural mechanics. The efficiency of a program depends largely on the numerical techniques employed and on their effective computer implementation. With regard to programming techniques, an optimum allocation of high and low speed storage is necessary.

A most important aspect of a general purpose computer program is, however, the ease with which it can be modified, extended and updated; otherwise, it may very well be that the program is obsolete within a few years after completion. This is because new structural elements are developed, better numerical procedures are available, or new computer equipment which requires new coding techniques is produced.

The structural analysis program SAP was designed to be modified and extended by the user. Additional options and new elements may easily be added. The program has the capacity to analyze very large three-dimensional systems; however, there is no loss in efficiency in the solution of smaller problems. Also, from the complete program, smaller special purpose programs can easily be assembled by simply using only those subroutines which are actually needed in the execution. This makes the program particularly usable on small size computers.



The current program version SAP IV for the static and dynamic analysis of linear structural systems is the result of several years' research and development experience. The program has proven to be a very flexible and efficient analysis tool. The program is coded in FORTRAN IV and operates without modifications on the CDC 6400, 6600 and 7600 computers. The first version of program SAP was published in September 1970 [28]. An improved static analysis program, namely SOLID SAP, or SAP II, was presented in 1971 [29]. Work was then started on a new static and dynamic analysis program. The program SAP III for static and dynamic analysis was released towards the end of 1972, but only to those agencies which supported our research. In relation to SAP III, the current version SAP IV has improvements throughout, and in particular has available a new variable-number-nodes thick shell and three-dimensional element, and out-of-core direct integration for time history analysis.

The structural systems to be analyzed may be composed of combinations of a number of different structural elements. The program presently contains the following element types:

- (a) three-dimensional truss element,
- (b) three-dimensional beam element,
- (c) plane stress and plane strain element,
- (d) two-dimensional axisymmetric solid,
- (e) three-dimensional solid,
- (f) variable-number-nodes thick shell and three-dimensional element,
- (g) thin plate or thin shell element,
- (h) boundary element,
- (i) pipe element (tangent and bend).

These structural elements can be used in a static or dynamic analysis. The capacity of the program depends mainly on the total number of nodal points in the system, the number of eigenvalues needed in the dynamic analysis and the computer used. There is practically no restriction on the number of elements used, the number of load cases or the order and bandwidth of the stiffness matrix. Each nodal point in the system can have from zero to six displacement degrees of freedom. The element stiffness and mass matrices are assembled in condensed form; therefore, the program is equally efficient in the analysis of one-, two- or three-dimensional systems.

The formation of the structure matrices is carried out in the same way in a static or dynamic analysis. The static analysis is continued by solving the equations of equilibrium followed by the computation of element stresses. In a dynamic analysis the choice is between

1. frequency calculations only,
2. frequency calculations followed by response history analysis,
3. frequency calculations followed by response spectrum analysis,
4. response history analysis by direct integration.

To obtain the frequencies and vibration mode shapes solution routines are used which calculate the required eigenvalues and eigenvectors directly without a transformation of the structure stiffness matrix and mass matrix to a reduced form. In the direct integration an unconditionally stable integration scheme is used, which also operates on the original structure stiffness matrix and mass matrix. This way the program operation and necessary input data for a dynamic analysis is a simple addition to what is needed for a static analysis.

The purpose in this part of the report is to present briefly the general program organization, the current element library and the numerical techniques used. The different options available for static and dynamic analyses are described and typical running times are given. In the presentation, emphasis is directed to the practical aspects of the program. For information on the development of the structural elements and the numerical techniques used the reader is referred to appropriate references.