SPHERICAL CONCRETE WATER TANK DESIGN

A Thesis

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Master of Science

by

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This thesis, by Steven James Hoggan, is accepted in its present form by the Department of Civil Engineering Science of Brigham Young University as satisfying the thesis requirement for the degree of Master of Science.

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Chapter I

INTRODUCTION

In the rapidly expanding world today there is an ever-increasing need to preserve and store water. This will be either fresh water pumped from wells or purified water coming from a treatment facility. Whichever the case, the water must be stored and available when needed by man or industry. Thus there is an extreme need for large, efficient, and economical storage facilities.

The solution to this problem is not trivial. The intent of this thesis is to suggest that a possible solution to this problem is the use of a thin shell. A thin shell is a curved surface or shell whose thickness is relatively small compared with its other dimensions and its radii of curvature. In reference to an egg; its shell is approximately 0.025 inches thick and its length is approximately 2.25 inches which yields a ratio of 0.011. Now consider a spherical dome with a radius of 100 feet and a design thickness of 3 inches. Its ratio is 0.0025. A considerable difference, but yet the dome is structurally sound. Therefore, there are many advantages to a thin shell. First and foremost is its inherent shape which yields strength and stability. This means that large quantities of materials are not needed to obtain structural soundness. A dome also has a certain aesthetic beauty because of its smooth, rounded surface.

With modern technology, the concrete thin shell is becoming more economical to construct. In the past, it has been set aside because of the high cost of forming, but in recent years this has changed because of a balloon technique. This new system makes use of an inflatable balloon which acts as the form for the dome. A urethane foam is then sprayed on the inside of the balloon which stiffens the system enough to tie the reinforcing steel. The concrete is placed by shotcrete methods. After completion, the balloon can be removed and reused. The urethane is left in place as it serves as insulation for the structure.

Objective

Today water tanks have taken on many shapes and configurations. Most common among these is a cylindrical tank with a roof of either a one or two-way slab or a partial spherical dome. Construction material has been either steel or reinforced concrete. Another shape commonly used, but mostly with steel construction, is that of a sphere.

The intent of this thesis is to determine the feasibility of using reinforced concrete in the design of a spherical water tank from which suggestions will be given for the best design. A mathematical solution using thin shell equations will describe the forces in the shell. A fortran program will be made of this mathematical solution for future use. A finite element solution will then be made to verify the results obtained from the mathematical approach. Also, a cost comparison will be made of a cylindrical versus a spherical tank of equal volumes.

Definition of Terms

The following definition of terms and symbols used frequently in thin shell analysis will also be used in this thesis.

1. Phi (ϕ): the angle measured from the edge of the dome to a point on the dome.

2. Psi (ψ): the angle measured from the edge of the dome to a point on the dome.

3. Alpha (α): the total angle of the dome from the apex to the edge of the dome.

4. Half sphere: a portion of a spherical dome not exceeding a total angle alpha of 90°.

5. Full sphere: referred to in this text as a spherical dome with a total angle alpha of more than 90°.

6. N'_{ϕ} : the radial stress component in the shell.

7. N'_{α} : the hoop stress component in the shell.

8. Stress resultant: a force per unit length described by thin shell equations.

Chapter II

ANALYSES

Mathematical Shell Analysis

The equations used in this analysis are those common to thin shell structures. Their applicability, of course, is dependent on the engineer's experiences and judgment. A constant thickness of a shell will be used to simplify both the mathematical and finite element analysis. However, in the actual design of a thin shell concrete sphere, it is desirable to vary the thickness of the shell from apex to edge.

A design will be made from the forces and conditions determined from the thin shell equations. The geometry of this simple design will then be coded and used in the finite element analysis. These results, which shall show the variations, should be, within reason, the stresses found in the initial mathematical design.

The analysis of a shell of revolution is divided into four parts(Billington,2).

1. A primary system consisting of the membrane solution.

2. The calculation of the errors due to membrane stress resultants (the rotation and horizontal movement of

shell edge).

3. Determine the corrections due to the unit loads at the boundaries.

4. Compatibility is accomplished by computing the size of the edge effects necessary to eliminate the errors.

The thin shell equations used in this analysis are now to be presented. Each equation will be given with some of the conditions and the limitations of use. All stress resultants will follow the usual sign convention, i.e. negative (-) is compression and positive (+) is tension.

Fixed-Edge Analysis

Primary System. Consider the membrane solution for the following conditions.

1. Dead load: uniform load over the surface (Figure 1).

2. Live load: uniform horizontally projected load over surface (Figure 2).

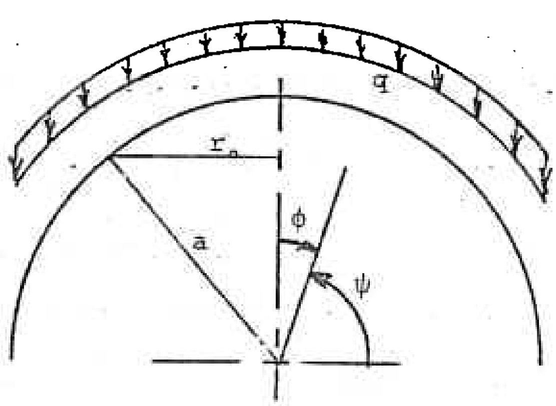


Figure 1. Dead Load

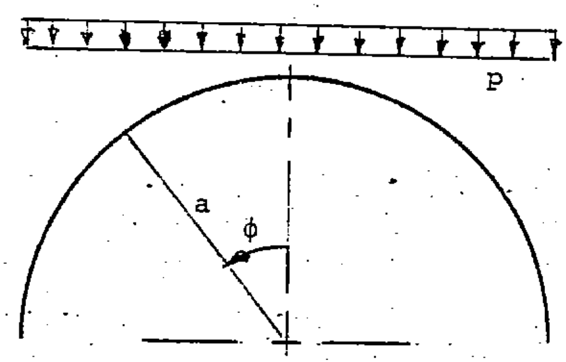


Figure 2. Live Load

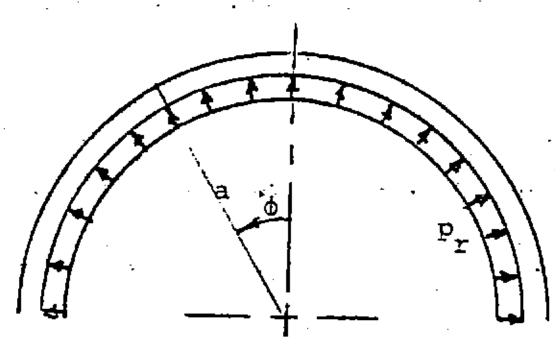


Figure 3. Constant Internal Pressure

3. Pressure load: internal [$+p_r$] or external [$-p_r$] pressure (Figure 3).

4. Fluid pressure: pressure increasing with depth of fluid (Figure 4).

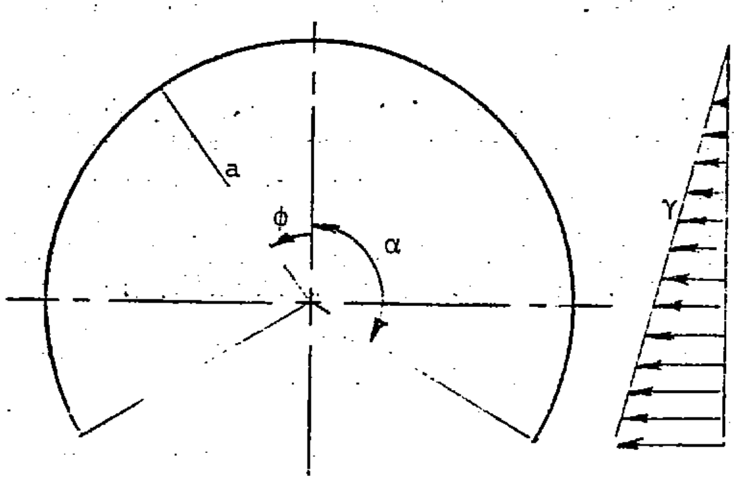


Figure 4. Fluid Pressure

A membrane condition is one in which the shell is considered to be free to move at its edges (Figure 5). It also assumes that there is no bending in the shell. The membrane solution is a straightforward and relatively simple solution for a given geometry. It can also provide a reasonable basis for design, but is subject to the following limitations (Billington,2).

1. The displacements due to membrane stress resultants can not give rise to appreciable bending in the shell.

2. The loading is distributed smoothly over the surface of the shell.

3. The boundary conditions must be such to supply

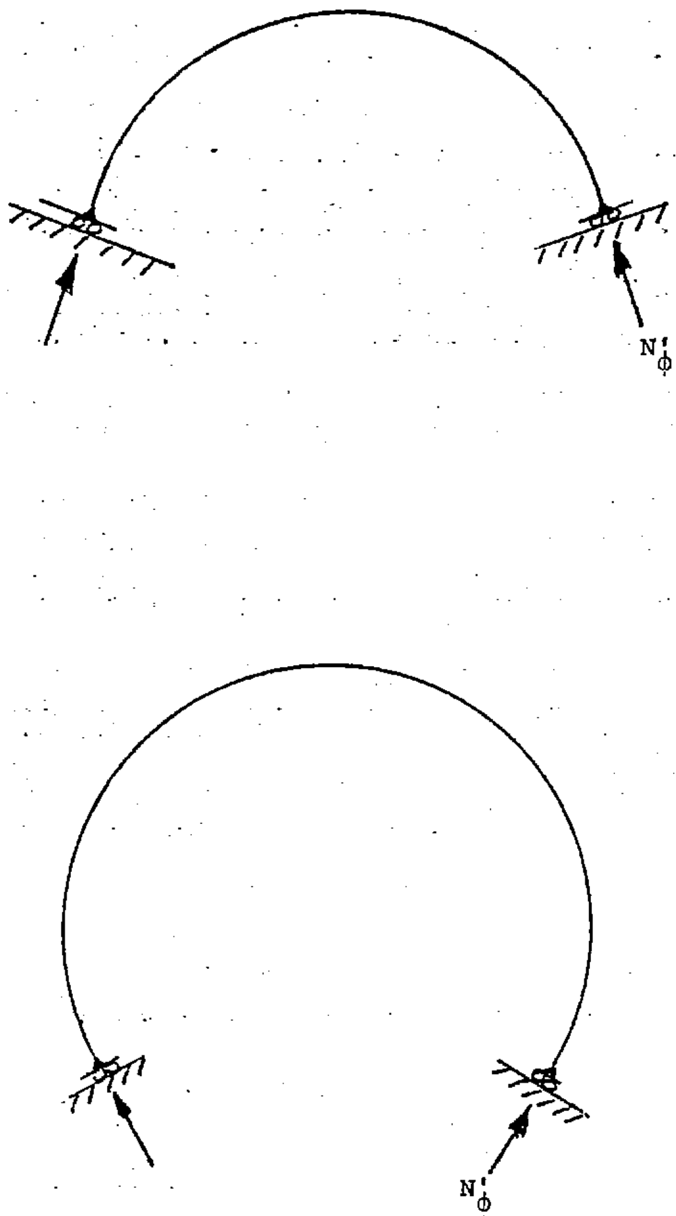


Figure 5. Support Condition for Membrane Theory

the forces and permit the displacements required by the membrane stress resultants.

A very detailed discussion of the limitations of the membrane theory can be found in Goldenveizer (6:474).

When considering the dead load (Figure 1) of the shell, the stress resultants become:

$$N'_\phi = -aq \frac{1}{1 + \cos\phi}$$

$$N'_\phi = aq \left(\frac{1}{1 + \cos\phi} - \cos\phi\right)$$

a = radius of curvation of shell (ft)

q = dead load (psf)

α = angle measured from shell apex

Figure 6 shows the distribution of these stress resultants over a sphere. It can be noted that at approximately 51° the hoop stresses become tension.

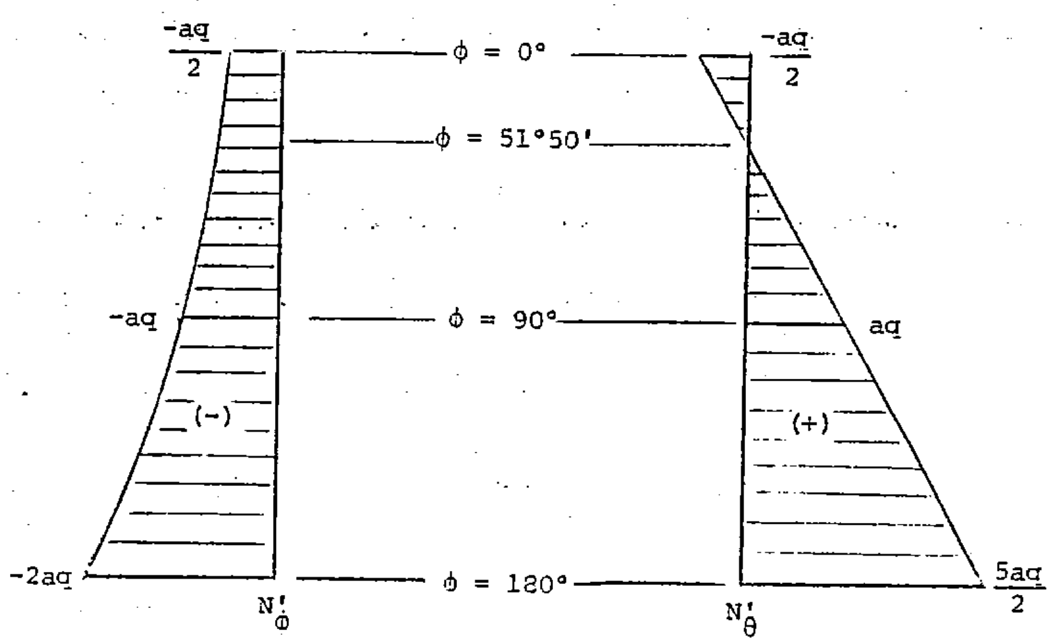


Figure 6. Distribution of Stresses for Dead Load

When considering a live load (Figure 2) application, the stress resultants are:

$$N'_{\phi} = -\frac{ap}{2}$$

$$N'_{\theta} = -\frac{ap}{2}\cos 2\phi$$

p = live load (psf)

Figure 7 shows the distribution of these stress resultants over a sphere. It can be noted that at 45° the hoop stresses become tension.

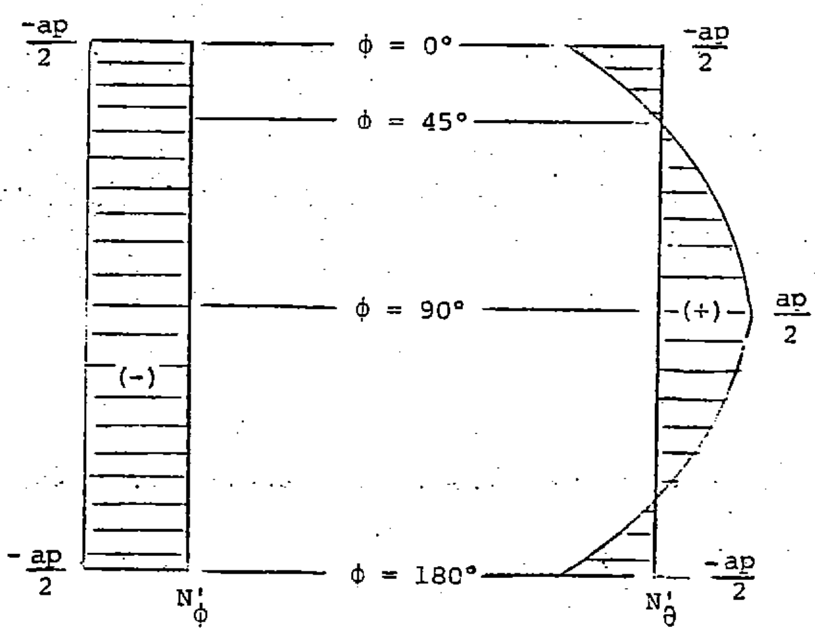


Figure 7. Distribution of Stresses for Live Load

When considering an internal pressure (Figure 3), the stress resultants become:

$$N'_\phi = \frac{a\,p_r}{2}$$

$$N'_\theta = \frac{a\,p_r}{2}$$

p_r = pressure (psf)

Figure 8 shows the distribution of these stress resultants over a sphere. The stresses induced by this loading condition add a simple constant stress throughout the shell.

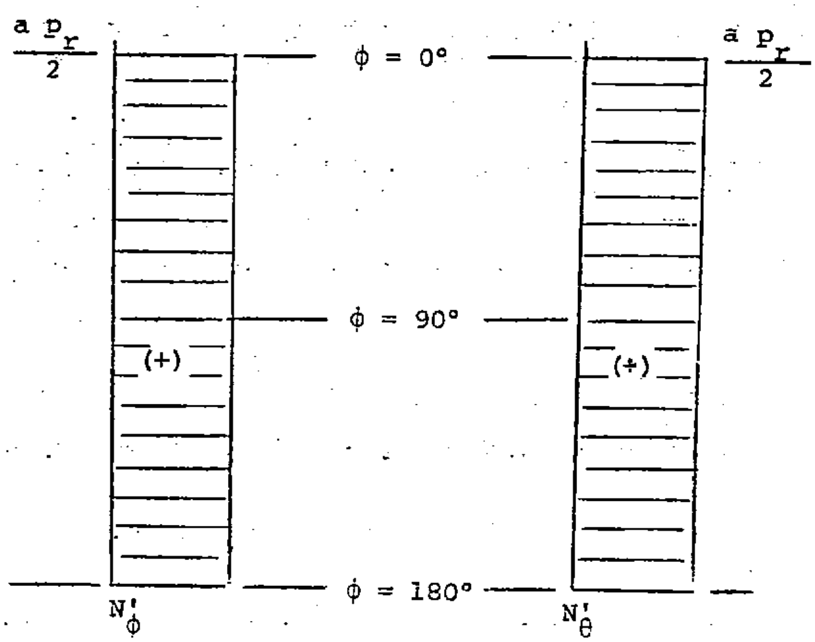


Figure 8. Distribution of Stresses for a Constant Internal Pressure

When considering a fluid pressure (Figure 4), the stress resultants become (Flugge,5:32):

$$N'_\phi = \frac{\gamma a^2}{6}\left(\frac{1 - \cos\phi}{1 + \cos\phi}\right)\left(1 + 2\cos\phi\right)$$

$$N'_\theta = \frac{\gamma a^2}{6}\left(\frac{1 - \cos\phi}{1 + \cos\phi}\right)\left(5 + 4\cos\phi\right)$$

γ = unit weight of fluid (pcf)

Figure 9 shows the distribution of stress resultants over a sphere. It is important to note here, that as a theoretical support gets closer to the bottom of the tank, the stresses N'_ϕ and N'_θ approach infinity, but in opposing directions. The radial stresses are approaching infinite compression while, transversly, the hoop stresses are approaching infinite tension.

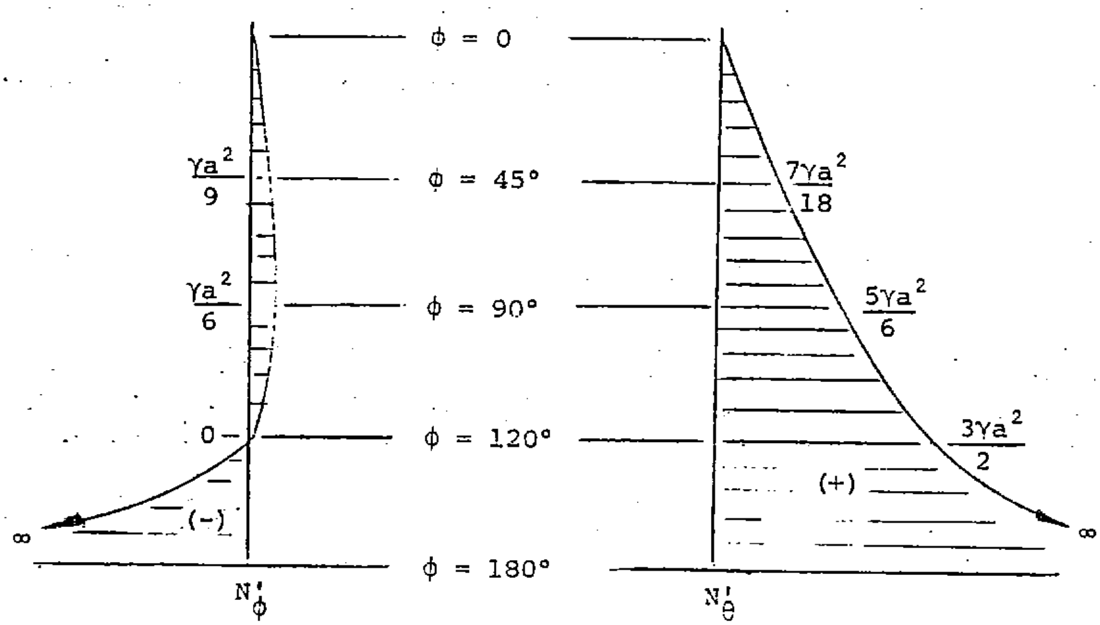


Figure 9. Distribution of Stresses for Fluid Pressure

Errors. When the loadings from the membrane condition are evaluated, it can be observed that there will be horizontal and rotational deformations. Thus it is necessary to calculate those deformations. The general equations which can be used for any membrane condition upon substitution of the proper membrane equations are:

$$\Delta_H = D_{10}^{D} = \frac{\cot\phi}{r_1 Eh}\left[N'_\phi(r_1 + \nu r_2) - N'_\theta(r_2 + \nu r_1)\right] - \frac{d}{r_1 d\phi}\left(\frac{\Delta_H}{\sin\phi}\right)$$

$$\Delta_\phi = D_{20}^{D} = \frac{r_2 \sin\phi}{Eh}(N'_\theta - \nu N'_\phi)$$

r_1 = a (for a shell of revolution)

r_2 = a (for a shell of revolution)

E = Young's modulus of elasticity

ν = Poisson's ratio

h = thickness of shell (may be modified in next section)

Δ_H = horizontal movement of shell edge

Δ_ϕ = rotation of shell edge

D_{10}^{D} = horizontal deformation due to membrane stress resultants

D_{20}^{D} = rotation due to membrane stress resultants

Thus, the equations for horizontal displacement and rotation for any given membrane condition can be formed.

For the constant shell thickness or uniform load over the dome surface, the errors are:

$$\Delta_H = D_{10}^{D} = \frac{a^2 q}{Eh}\left(\frac{1+\nu}{1+\cos\phi} - \cos\phi\right)\sin\phi\ ;$$

$$\Delta_\phi = D_{20}^{D} = -\frac{aq}{Eh}(2+\nu)\sin\phi$$

For a uniform horizontally projected load, the errors become:

$$\Delta_H = D_{10}{}^D = \frac{a^2 p}{2Eh} (\nu - \cos 2\phi) \sin \phi$$

$$\Delta_\phi = D_{20}{}^D = - \frac{ap}{2Eh} (3 + \nu) \sin 2\phi$$

For a uniform internal pressure, the errors become:

$$\Delta_H = D_{10}{}^D = \frac{a^2 p_r}{2Eh} \sin \phi \ (1 - \nu)$$

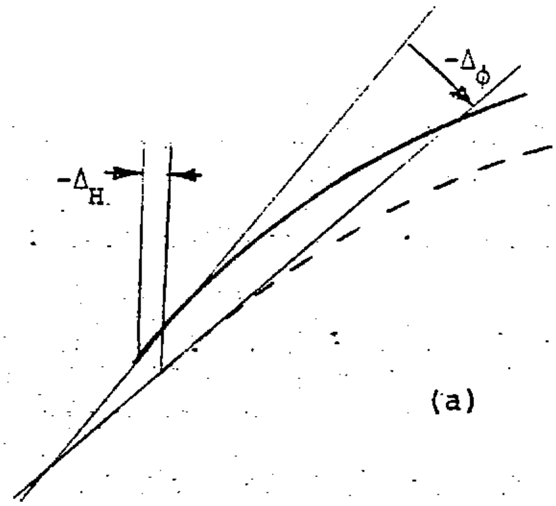
$$\Delta_\phi = D_{20}{}^D = 0$$

For an internal fluid pressure, these errors become:

$$\Delta_H = D_{10}{}^D = \frac{\gamma a^3}{6Eh} \sin \phi \left(\frac{1 - \cos \phi}{1 + \cos \phi}\right)\left(5 + 4 \cos \phi - \upsilon - 2\upsilon \cos \phi\right)$$

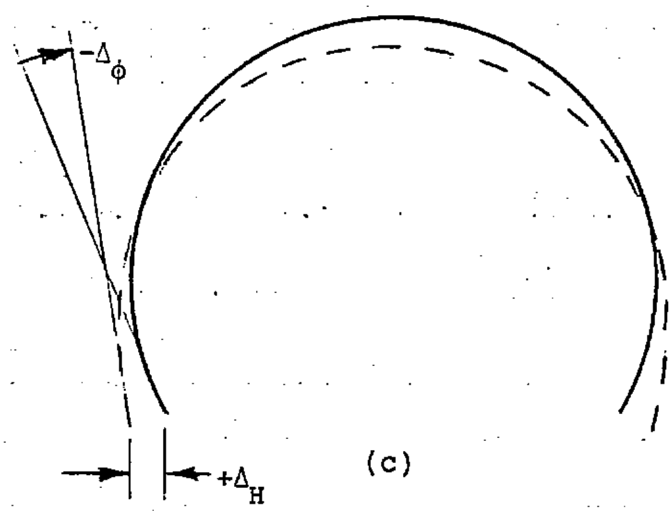
$$\Delta_\phi = D_{20}{}^D = - \frac{\gamma a^2}{Eh} \sin \phi$$

<u>Sign Convention.</u> The sign convention for the deformations due to the membrane stress resultants are shown in Figure 10. An outward horizontal movement of the shell edge is considered positive. When considering the left edge of the shell, a counterclockwise rotation of that edge is considered positive.



(a)

(b)



(c)

Figure 10. (a) Shell Deformations due to Membrane Stress Resultants (b) Deformed Shape Under Membrane Loading (c) Deformed Shape and Deformations that Result from Membrane Loadings of Water Tank

Corrections. It is now necessary to calculate unit forces that will be required to correct the deformations that took place due to the membrane stress resultants. It is also necessary to consider the effects of bending.

In the considerations for bending and the derivation of the equations, many assumptions have been made. One consideration is that just as bending was neglected in solving for membrane stress resultants, now membrane stress resultants will be neglected when calculating the bending effects. It is also assumed that the edge effects are rapidly damped oscillating functions. Further assumptions can be found in Billington (2:59-65).

Also, an approximate solution to determine the edge effects will be used because of the complexity of the exact solution. It must be noted that this approximation also has some limitations. It does not apply where there is some other discontinuity in the shell such as a skylight opening in the top. However, if the solution gives negligible displacements and stresses at the opening, then the approximation can be considered valid. The derivation of these equations can be found in Timoshenko (7).

In applying a uniform unit moment, M_α, at the shell edge the results are:

$$\Delta_H = D_{21}{}^D = \frac{2\lambda^2 \sin\alpha}{Eh} M_\alpha$$

$$\Delta_\alpha = D_{22}{}^D = \frac{4\lambda^3}{Eha} M_\alpha$$

and similarly applying a uniform horizontal unit force, H, at the shell edge:

$$\Delta_H = D_{11}{}^D = \frac{2a\lambda \sin^2 \alpha}{Eh} H$$

$$\Delta_\alpha = D_{12}{}^D = \frac{2\lambda^2 \sin \alpha}{Eh} H$$

where:

$$\lambda^4 = 3(1 - \nu^2)\left(\frac{a}{h}\right)^2$$

It can be noted that for $H = M_\alpha$ then $\Delta_\alpha = \Delta_H$, this follows because of the reciprocity theorem.

In applying these unit forces at the edge of the shell, then forces are also induced into the shell. They are:

$$N_{\phi_1} = -\sqrt{2} \cot(\alpha - \psi)(\sin \alpha)(e^{-\lambda\psi}) \sin(\lambda\psi - \frac{\pi}{4}) H$$

$$N_{\phi_2} = -\frac{2\lambda}{a} \cot(\alpha - \psi) e^{-\lambda\psi} \sin(\lambda\psi) M_\alpha$$

$$N_{\theta_1} = -2\lambda \sin \alpha \, e^{-\lambda\psi} \sin(\lambda\psi - \frac{\pi}{2}) H$$

$$N_{\theta_2} = -\frac{2\sqrt{2}\lambda^2}{a} e^{-\lambda\psi} \sin(\lambda\psi - \frac{\pi}{4}) M_\alpha$$

$$M_{\phi_1} = \frac{a}{\lambda} \sin \alpha \, e^{-\lambda\psi} \sin(\lambda\psi) H$$

$$M_{\phi_2} = \sqrt{2} e^{-\lambda\psi} \sin(\lambda\psi + \frac{\pi}{4}) M_\alpha$$

Sign Convention. For a positive M_α, there is a positive rotation and a positive (outward) translation and for an outward thrust, H, there is a positive rotation and a positive translation (Figure 11).

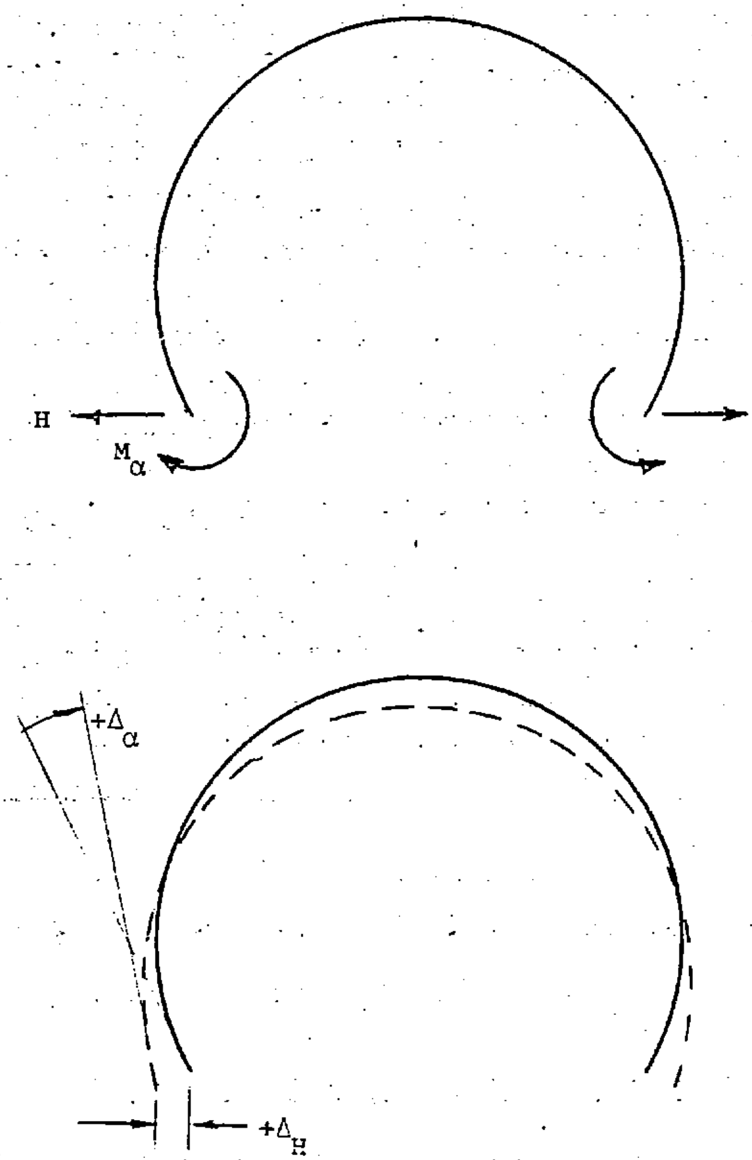


Figure 11. Deformations Caused by Unit Correction Forces Applied at the Shell Edge

The horizontal displacements found here are the same sign or direction as those found in the errors. However, the rotation considered positive here is not the same positive direction considered in the errors.

Therefore, when continuing from here to solve for the exact horizontal force and moment needed to correct the errors from the membrane solution, it will be necessary to change the sign of the actual rotation calculated in the errors.

<u>Compatibility.</u> All the necessary information is now available to solve for the total stress resultants. Since no consideration has been given to a ring, the solution will be a fixed dome solution.

The horizontal forces are summed and set equal to zero to obtain the exact correctional forces (thus a fixed dome). Likewise, the sum of the rotations must also be zero.

$$\Sigma\Delta_H = X_1 D_{11}{}^D + X_2 D_{12}{}^D + D_{10}{}^D = 0$$

$$\Sigma\Delta_\alpha = X_1 D_{21}{}^D + X_2 D_{22}{}^D + D_{20}{}^D = 0$$

where: X_1 = horizontal correction force

X_2 = moment correction force

$D_{10}{}^D$ = horizontal deformation due to membrane stress resultants

$D_{20}{}^D$ = rotation due to membrane stress resultants

$D_{11}{}^D$ = horizontal deformation due to horizontal unit load

$D_{12}{}^D$ = horizontal deformation due to unit moment

$D_{22}{}^D$ = rotation due to unit moment

These values of $X_1(H)$ and $X_2(M_\alpha)$ can now be multiplied by the previous equations and then added to the respective membrane condition to obtain the total stress resultants.

Shell-Ring Analysis

Membrane. The membrane solution is the same as in the previous section. The horizontal component of N'_ϕ at the edge is now to be held by the ring. The tension in the ring will be:

$$T = N'_\alpha r_0 \cos\alpha$$

where: r_o = a sin

N'_α = the radial stress component at shell edge

α = total angle of shell

It is important, however, to recognize that since there will be bending in the shell near the edge, that it will be necessary to increase the thickness in the shell as shown in Figure 12. This allows the steel to be placed so as to be more efficient. Since the shell will be thickened then this will also change the shell deformations as calculated previously.

Therefore, the assumption is made that the thickness of the shell is constant and equal to the actual thickness, h, at a certain distance from the edge (Figure 12).

$$S_a = 0.5\sqrt{ah_a}$$

where: S_a = the distance from the shell edge to the design thickness

h_a = numerical average thickness of thickened region

It is also assumed that the total length of the thickened region is:

$$S = 2\sqrt{ah_a}$$

It is important to note that as the thickness of the shell increases, the dome will be "stiffer" than the ring, and hence, would take more of the horizontal thrust by hoop tension. Likewise, if the ring was "stiffer," by comparison to the dome, then the hoop tension and bending moments in the dome become smaller, but the tension in the ring increases.

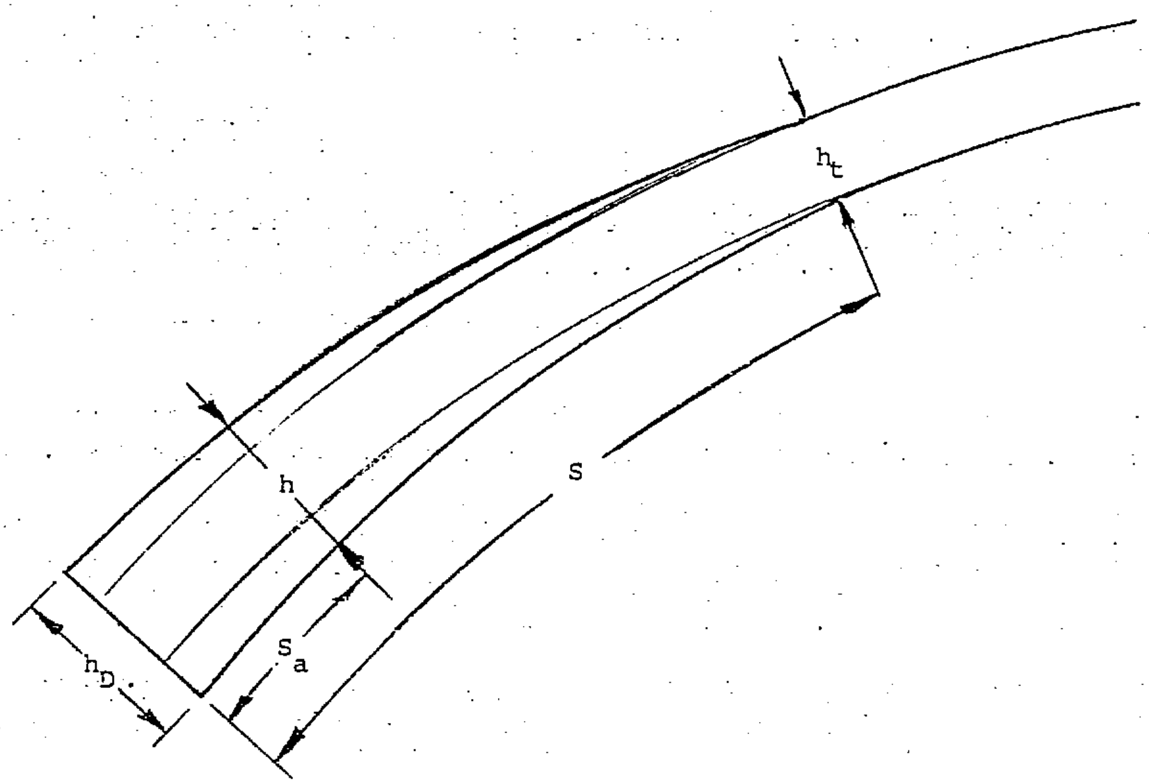


Figure 12. Thickened Portion of Shell

Errors. When including a ring, the errors involve four deformations at the shell edge:

1. Horizontal displacement of dome.
2. Rotation of dome.
3. Horizontal displacement of ring.
4. Rotation of ring.

The first two, which depend on shell loading, were given in the fixed-edge analysis section. The ring deformations due to the membrane stress resultants at the shell edge are:

$$\Delta_H = D_{10}{}^R = \left(\frac{\cos \alpha}{A_R} + \frac{y_o e}{I_R} \right) \frac{r_o^2}{E} N'_\alpha$$

$$\Delta_\alpha = D_{20}{}^R = -\frac{r_o e}{EI_R} N'_\alpha$$

I_R = moment of inertia of ring

A_R = area of ring

e = eccentricity of N'_α on ring

The eccentricity can be calculated for a half sphere dome by:

$$e = (y_0 - (b/2 - d'\tan\alpha)\tan\alpha)\cos\alpha$$

and for a full sphere a mirror angle (180 − α) needs to be used with the following equation:

$$e = ((b/2 - d'\tan\alpha)\tan\alpha - y_0)\cos\alpha$$

where:

$$d' = \frac{h_D}{2}\cos\alpha$$

$$y_0 = \frac{d}{2} - d'$$

Sign Convention. The sign convention for these deformations are summarized in Figure 13. N'_α is taken as negative (compression) and e is positive, when applied to the ring, as shown in Figure 13. Inward translation and counterclockwise rotation are taken as positive. Therefore when N'_α is in compression; hence negative, a negative (outward) translation and a positive (counterclockwise) rotation on the ring occurs, as shown in Figure 14.

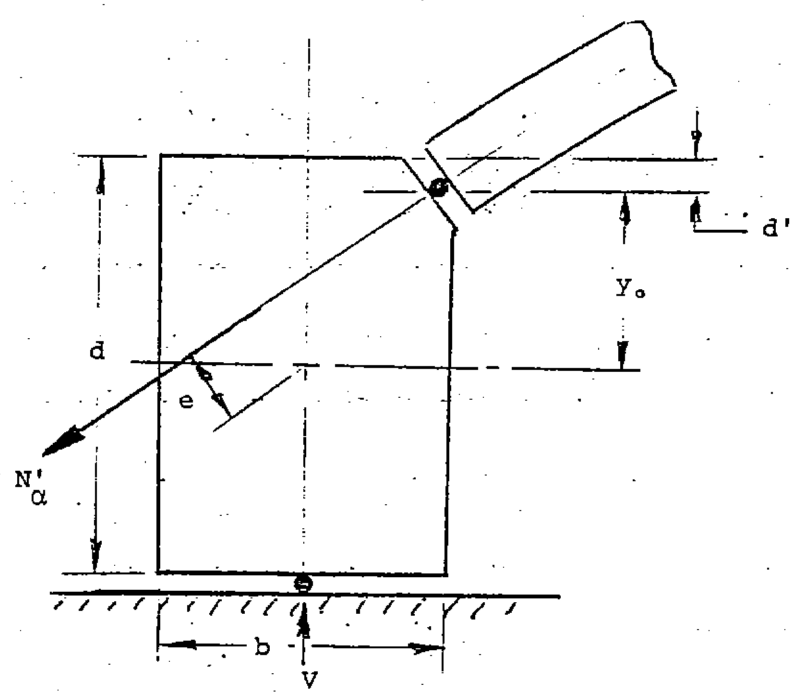


Figure 13. Shell-Ring Connection

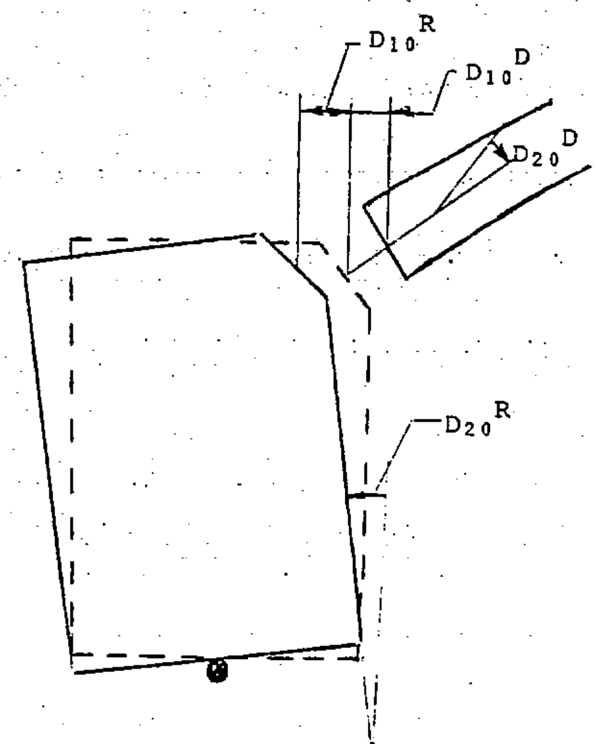


Figure 14. Shell-Ring Deformations

Corrections. When a unit force is placed on the ring at y_o then the ring horizontal displacements become:

$$\Delta_H = D_{11}^R = \left(\frac{1}{A_R} + \frac{y_0^2}{I_R}\right)\frac{r_0^2}{E}$$

$$\Delta_\phi = D_{21}^R = -\frac{r_0^2 y_0}{EI_R}$$

and when a unit moment is placed on the ring then the ring rotations become:

$$\Delta_H = D_{12}^R = -\frac{r_0^2 y_0}{EI_R} = D_{21}^R$$

$$\Delta_\phi = D_{22}^R = \frac{r_0^2}{EI_R}$$

The ring corrections are now added to the dome corrections to obtain the total corrections.

$$D_{11} = D_{11}^D + D_{11}^R$$

$$D_{12} = D_{12}^D + D_{12}^R$$

$$D_{22} = D_{22}^D + D_{22}^R$$

Compatibility. The compatibility equations are written as before and solved for the exact corrections needed to satisfy compatibility.

Ring Tension. It should be noted that from the membrane theory the ring is in tension and the dome is in compression. From membrane theory, the ring tension, as previously noted, is:

$$T = N'_{\phi}\, r_o \cos\alpha$$

The dome movement is outward but not as far as predicted by membrane theory (Δ_H) because of the restraint of the dome. Thus, the ring tension is reduced by the correction force X_1, and dome tension occurs and some bending. Figures 15 and 16 show free body diagrams of shell rings for the cases of a dome roof and a water tank respectively. The figures show the directions of each correction force for the given shell. Taking the sign convention used in the analysis and substituting X_1 and N'_{α} (with their respective signs from the equations) then the ring tension is defined by:

$$T = (-N'_{\alpha} \cos\alpha - X_1)\, r_0$$

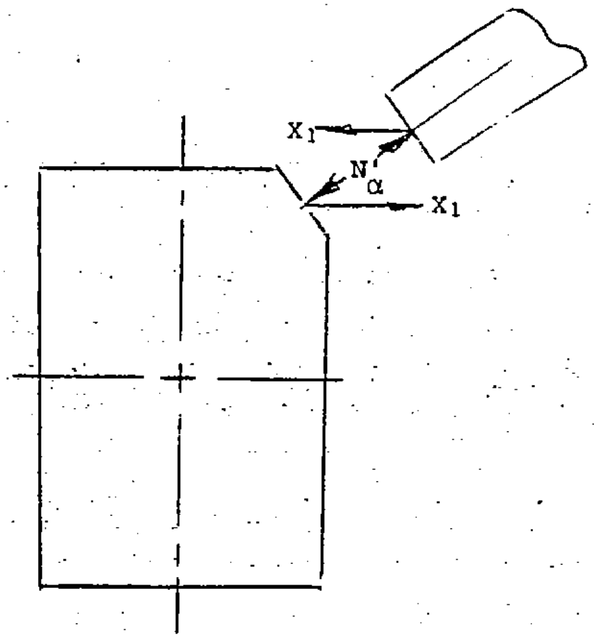


Figure 15. Free-Body Diagram for Shell-Ring of a Partial Sphere

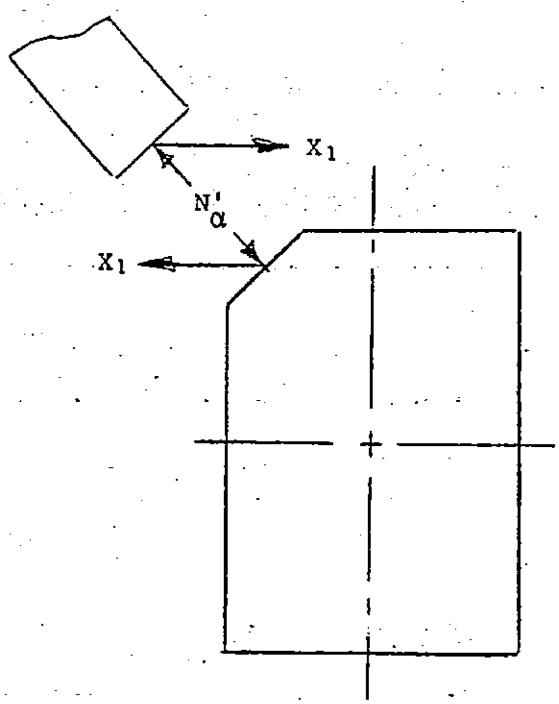


Figure 16. Free-Body Diagram for Shell-Ring of a Full Sphere

Chapter III

DESIGN AND RESULTS

The chosen problem has the following parameters:

$$a = 40 \text{ ft}$$

$$\alpha = 120°$$

$$h = 12 \text{ in (constant)}$$

and the following concrete properties:

$$f'_c = 3{,}000 \text{ psi}$$

$$\text{unit weight} = 150 \text{ pcf}$$

$$\nu = 0.2$$

the loading conditions are:

dead load = 150 psf

live load = 30 psf

unit weight of water = 62.4 pcf

the attached ring beam is:

$$b = 18 \text{ in}$$

$$d = 24 \text{ in}$$

The final stress resultants are shown in Table I. These stress resultants can now be used to determine the necessary steel requirements, thus a complete design is accomplished.

In order to compare costs of tanks of equal volumes, it will be necessary to use the following geometry to obtain a total volume of 2 million gallons.

TABLE I

STRESS RESULTANTS FROM MATHEMATICAL ANALYSIS WITH DEAD AND LIVE LOAD AND FLUID PRESSURE

Angle From Dome Apex	N-Phi Radial. Kips/ft	N-Theta Hoop Kips/ft	M-Phi Moment Ft-Kip/ft
120.0	-8.636	38.998	-3.806
119.0	-8.108	55.212	-7.872
118.0	-7.424	70.437	-10.640
117.0	-6.615	84.276	-12.335
116.0	-5.709	96.482	-13.164
115.0	-4.735	106.930	-13.315
110.0	0.360	133.930	-8.860
105.0	4.601	131.528	-3.335
90.0	10.040	90.295	0.478
60.0	6.494	40.144	-0.004
30.0	-0.551	7.831	-0.000
0.0	-3.600	-3.600	0.000

$a = 42.25$ ft

$\alpha = 120°$

$h_t = 6$ in (top)

$h_d = 12$ in (edge)

The detailed calculation for this water tank can be found in Appendix A. It also includes steel calculations, quantities, and a design drawing. The required quantities of material for this tank are:

volume of concrete = 450 cubic yards

reinforcing steel = 346,730 pounds

Cost estimate:

Concrete in place @ \$200 per cu-yd	= \$ 90,000
Steel in place @ \$.40 per pound	= \$138,692
Total Cost	= \$228,692

A 2 million gallon cylindrical tank recently built required 1,260 cubic yards of concrete and resulted in a total cost of \$239,000 (\$190 per cu. yd., which included the cost of the steel). Chapter V gives an alternative design that reduces the cost of the spherical tank.

Chapter IV

DISCUSSION OF RESULTS

Comparison

In order to better compare the results from the mathematical and the finite element analysis, it will be necessary to modify the results of the mathematical analysis to show only the stresses due to the fluid pressure. The previous mathematical analysis included dead load and live load which were not included on the finite element analysis. This modification is shown in Table III.

The units from the finite element analysis are pounds per square foot. To make these equivalent to those of the mathematical analysis, it is necessary to multiply by the thickness of the shell to obtain a force per unit length. Since the thickness is 12 inches, or one foot, then there is no numerical change.

Upon inspection of Table IV, it can be seen that the stresses are very near the same. This shows that, at least at the critical points, the two methods of analysis are good. It should be kept in mind that the finite element solution is an approximate solution, but the mathematical analysis used is also an approximate solution. The latter has been proven to be a good approximate solution (Timoshenko,7).

TABLE III

STRESS RESULTANTS FROM MATHEMATICAL ANALYSIS
FOR ONLY FLUID PRESSURE

Angle From Dome Apex	N-Phi Radial Kips/ft	N-Theta Hoop Kips/ft	M-Phi Moment Ft-Kip/ft
120.0	2.422	70.146	-3.087
119.0	3.074	79.848	-5.557
118.0	3.800	88.864	-7.211
117.0	4.582	96.956	-8.193
116.0	5.402	103.978	-8.636
115.0	6.244	109.863	-8.659
110.0	10.303	123.227	-5.617
105.0	13.419	118.153	-2.063
90.0	16.640	83.484	0.305
60.0	11.094	38.838	-0.003
30.0	3.264	10.112	0.000
0.0	0.000	0.000	0.000

TABLE IV

STRESS RESULTANTS: FINITE ELEMENT ANALYSIS VERSUS MATHEMATICAL ANALYSIS

	Finite Element Analysis (Average)	Mathematical Analysis
Radial Stress At $\phi = 90°$	17,000 lbs/ft	16,400 lbs/ft
Hoop Stress At $\phi = 105°$	107,100 lbs/ft	118,153 lbs/ft
Tension In Ring Beam	(free) 270 kips average 158 kips (fixed) 45 kips	167 kips

Chapter V

CONCLUSIONS

Summary of Possible Designs

As a result of the research conducted, the following designs are possible.

1. The first possible solution could be to use full sphere that is partially buried (Figure 36). The advantage of this design is that some of the stresses can be offset by earth pressures. If the tank was completely buried, then it could possibly be pressurized to obtain a required head. The major disadvantage is that a water test will be required before backfilling the earth, thus requiring steel as if it were not buried.

2. Another solution would be to stop the sphere at $\phi = 120°$ (Figure 37). This point is chosen as it reduces the radial stresses to either zero or compression. Also, cutting off the bottom does not decrease the volume of the tank significantly. This gives rise to the next design possibility.

3. This solution would have a sphere with a total angle of 120° as before but with a limited filling depth (Figure 38). This will decrease the head of water significantly while only increasing the overall diameter

Put in bookcase

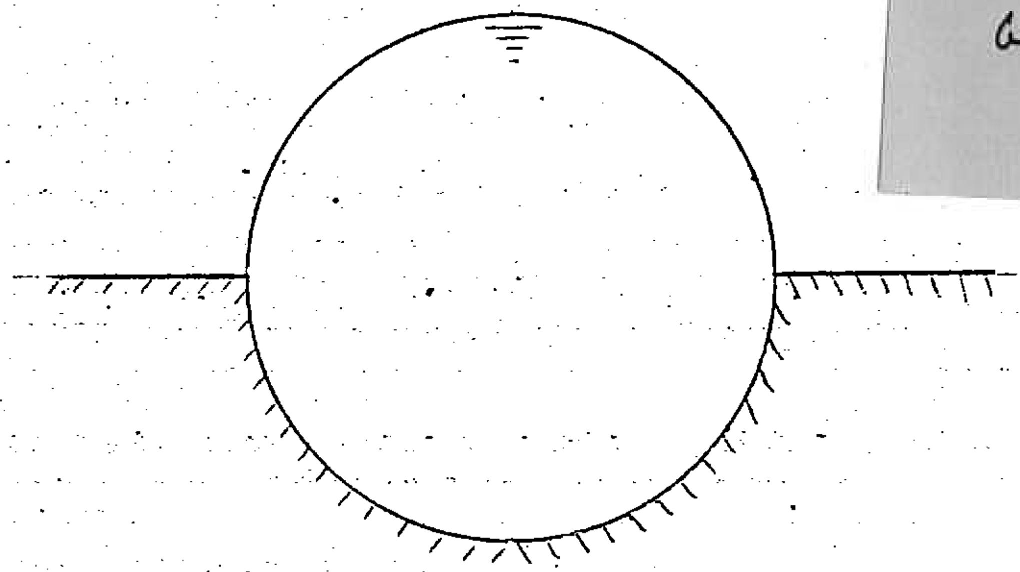


Figure 36. Full Sphere Water Tank Partially Buried

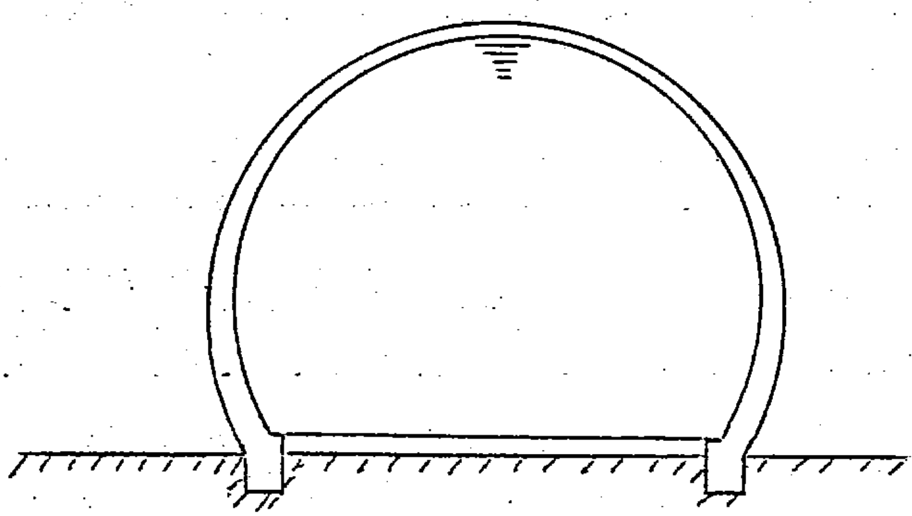


Figure 37. Partial Sphere Filled with Water

slightly. As an example, consider the two million gallon tank previously designed. The radius would be increased from 42.25 feet to 45 feet. The total height of the tank would increase from 63 feet to 68 feet. But instead of filling the tank, only 46 feet of water would be needed to obtain the 2 million gallons. Therefore, this represents a loss of 63 - 46 = 17 feet of water head. As previously noted, the critical hoop stress occurs at $\phi = 105°$, the stress at that point due only to the water pressure can be calculated using the following equation which allows a variable depth of water (Fischer,4:157).

$$N'_\theta = \frac{\gamma a^2}{6 \sin^2\phi} \left(4\cos^3\phi - 3\cos\alpha \cos^2\phi - 6\cos\phi + 6\cos\alpha - \cos^3\alpha \right)$$

where: α = angle from shell apex to level of water

ϕ = angle from shell apex to point of desired stress

which at $\phi = 105°$ and $\alpha = 60°$ becomes

$$N'_\phi = 96{,}100 \text{ lbs/ft}$$

The stress from the 42.25 foot radius tank at that point was

$$N'_\phi = 125{,}000 \text{ lbs/ft}$$

This represents a 23 percent decrease in maximum stress. Let that represent a 23 percent reduction in reinforcing steel. The required volume of concrete is now 500 cubic yards. The total cost of the tank is now $206,792, which is a $21,899 savings over the previous design. This also saves a total of $32,208 over the cylindrical tank, which is a 13.5 percent decrease in the cost of the tank. (The

23 percent reduction in steel takes into account a larger circumference and a significant decrease of steel in the top portion of the sphere.)

Conclusions

The conclusions that can be reached as a result of the research conducted are:

1. It is possible to use a spherical reinforced concrete shell to store water.

2. The thin shell mathematical analysis presented herein provides reasonable stress resultants needed to design steel reinforcement. The mathematical analysis was verified by the finite element analysis.

3. A significant savings in total cost can be achieved as a result in less concrete and less reinforcing steel.

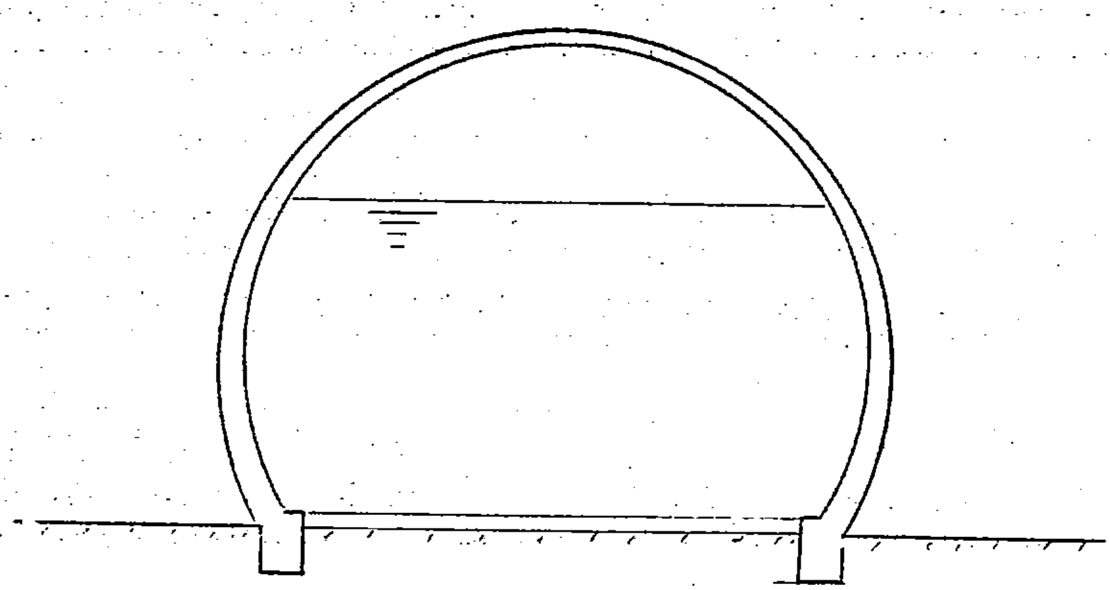


Figure 38. Partial Sphere Filled to a Maximum Level

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