

LARGE THIN SHELL CONCRETE DOMES
USING AIR SUPPORTED FORMS AND CABLE NETS

A Thesis
Submitted to
The Department of Civil Engineering
Brigham Young University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science

by
Robert J. Hatch
August 1994

This thesis, by Robert J. Hatch, is accepted in its present form by the Department of Civil Engineering of Brigham Young University as satisfying the thesis requirement for the degree of Master of Science.

Date

Chairman - Advisory Committee

Member - Advisory Committee

Chairman - Major Department

TABLE OF CONTENTS

<u>Chapter</u>	<u>Page</u>
LIST OF FIGURES	ii
LIST OF TABLES	iii
LETTER SYMBOLS	iv
NOTATION	v
CHAPTER 1 INTRODUCTION AND OBJECTIVE	1
OBJECTIVE	1
INTRODUCTION	1
CONTENTS	5
CHAPTER 2 CABLE NET SOLUTION	7
CABLE NET PATTERNS	8
THE SYSTEM	11
CHAPTER 3 RADIAL CABLE NET SYSTEM	14
RADIUS OF CURVATURE	14
AIR FORM SURFACE AREA	19
CABLE FORCES	21
CABLE LENGTHS	28
STABILITY	31
PROBLEMS AND LIMITATIONS	34
CHAPTER 4 GEODESIC CABLE NET SYSTEM	36
RADIUS OF CURVATURE	36
AIR FORM SURFACE AREA	40
INTRODUCTION TO CABLE FORCES	41
GEODESIC GEOMETRY	45
CABLE LENGTHS	60
CALCULATION OF CABLE FORCES	68
STABILITY	73
CABLE CONNECTIONS	74
CHAPTER 5 CONCLUSIONS	77
BIBLIOGRAPHY	79

LIST OF FIGURES

<u>FIGURE</u>		<u>PAGE</u>
1	A Typical Anchoring System for An Air Form	3
2	Radial Cable Net Pattern	9
3	Geodesic Cable Net Pattern	10
4	Radius of Curvature	17
5	Air Pressure Within Air Form	22
6	Cable Force Vectors	24
7	Small Circular Cable @ Apex	30
8	Cable Orientation	44
9	Icosahedron	46
10	6-Frequency Icosahedron	48
11	9-Frequency Icosahedron	49
12	15-Frequency Icosahedron	50
13	Geodesic Cable Pattern Icoscap	58
14	Combination of Icosa Faces	59
15	Typical Cable Connection	75

LIST OF TABLES

<u>TABLE</u>		<u>PAGE</u>
1	Radial Cable Net Calculations	16
2	Multiple-wire Strand Properties	26
3	Geodesic Cable Net Calculations	38
4	Chord Factors for 6-Frequency Icosahedron	54
5	Chord Factors for 9-Frequency Icosahedron	55
6	Chord Factors for 15-Frequency Icosahedron	56
7	Cable Lengths for Various Dome Sizes	65

LETTER SYMBOLS

in.	inch
ft.	feet
lb.	pounds
lb/in.	pounds per inch
lb/ft.	pounds per foot
k/ft.	kips per foot
kips	1000 pounds
mil	0.001 pounds
pcf	pounds per cubic foot
pg.	page
psi	pounds per square inch
psf	pounds per square foot
ft ² .	square feet

NOTATION

f	Frequency of icosahedron face
d	Geodesic chord factor
l	Reduced length of radial cable, ft.
P	Air pressure used to inflate the air form, psf
r	Horizontal dome radius at foundation, ft.
r_a	Radius of curvature of the air form, ft.
r_c	Horizontal radius of circular cable at apex, ft.
u	Uplift force on air form associated with a tributary length 'ring beam', ft.
w	Weight of a tributary length of foundation between cables along the dome perimeter, lb.
A	Surface area of 'skeleton' dome, ft ²
C	Ring compression on shell at circular rib, lb.
F	Cable tensile force, kips
H	Height of the dome, ft.
L	Length of each cable required, ft.
N	Force in the air-supported form (i.e. fabric), lb/ft.
NC	Number of cables required at foundation.
P	Load along perimeter of circular 'rib', lb/ft.
R	Radius of curvature of the cable net, ft.
R_c	Radius of curv. of the module at apex, ft.
U	Total uplift force on air form, kips
W	Total weight of the foundation system, kips

CHAPTER 1

INTRODUCTION AND OBJECTIVE

OBJECTIVE

The objective of this thesis is to show how steel cable nets can be incorporated into an air-supported forming system to facilitate the construction of large spherical concrete domes. The steel cable nets not only provide additional resistance to external forces, but assist in reducing the radius of curvature of air-supported forms. This thesis is written for an audience who may have limited background with domes but understand basic structural engineering principles. The structural analysis and design of concrete domes constructed with cable nets is beyond the scope of this thesis. Likewise, a finite element analysis will be required to complete the structural design of the proposed style of domes.

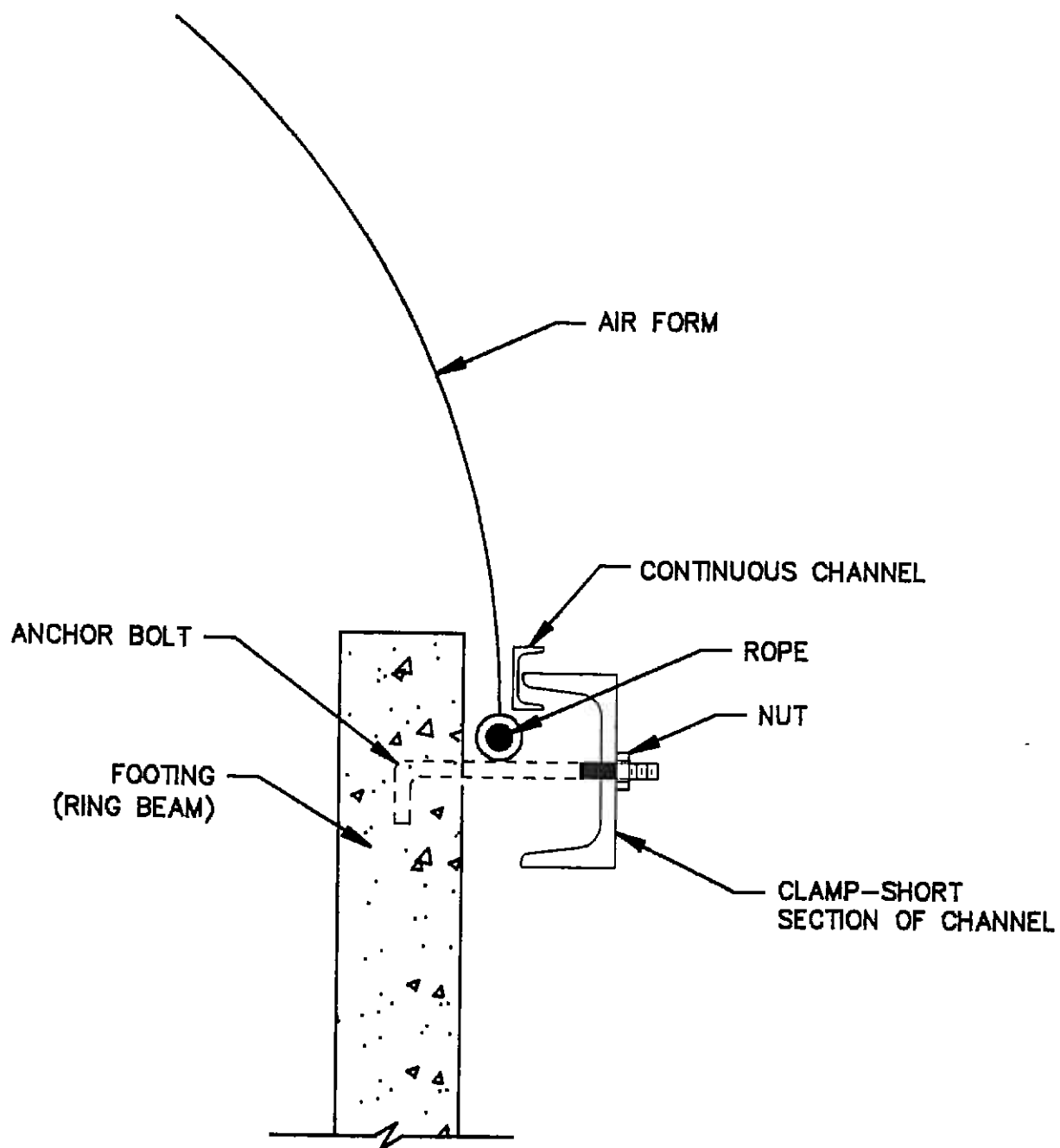
INTRODUCTION

The thin shell concrete dome has become a very popular and affordable structure with a wide variety of applications. Domes of this type have been constructed to store water, granular materials, mechanical equipment and fruit. Even residential homes, churches and schools have been constructed with this double curved structure. In fact, the double curvature of the dome is where its efficiency is derived. One is able to maximize space within a structure while minimizing the materials required to build it. Unfortunately, in the

early years of its outset, the cost of building the forms for a concrete dome easily doubled the cost of the project. However, due to developments within the last few decades, it is now possible to avoid the intense formwork required.

The new concept for structural forms involves the inflation of a flexible membrane form often called a 'balloon.' The air form is made out of such fabrics as nylon and polyester in order to meet the requirements of durability, strength, and shape. In order to create the spherical shape of an air form, triangular pieces of fabric are welded together at 1000 degrees Fahrenheit. These welds as well as the fabric must be capable of minimizing elongations as the air form is inflated. Before inflating the air form, it is attached to the foundation or 'ring beam' as shown in Figure 1. Once the air-supported form is inflated, there are several methods that have been developed for the placement of steel reinforcing and concrete.

In one such system, developed about twenty years ago, the dome construction actually occurs on the inside. For large diameter domes, the air form is typically inflated with 1 to 2 in. of water pressure using dual blowers and motors to provide a continuous supply of air. Once the air form is fully inflated, urethane foam is sprayed on the inside surface of the air form providing a means for attaching the reinforcing steel. In fact, the urethane foam will act as an insulation layer for the completed structure. Shotcrete is then applied to the inside surface in 1/2 in. thick layers



A TYPICAL ANCHORING SYSTEM FOR AN AIR FORM

FIGURE 1

after the reinforcing steel is in place. Starting at the base of the air form, care must be taken to ensure that the shotcrete is applied uniformly in order to prevent a local collapse.

Owing to its efficiency, over 500 concrete domes have been constructed with this system of air-supported forming in the United States as well as in various parts of the world. These domes vary in size from 14 to 260 ft. in diameter and range from spherical to elliptical. In addition to its cost effectiveness, the air-supported forming system provides the advantage of having a controlled atmosphere that permits year round construction.

Recently, there has been an interest to build large concrete domes in excess of 300 ft. in diameter (throughout this thesis, large domes will refer to sizes in excess of 300 ft. in diameter). Unfortunately, the air-supported forming system places an upper limit on the size of a thin shell concrete dome that can be safely constructed. The typical air form is only capable of withstanding a certain amount of air pressure before the allowable stresses in the fabric and the welds are exceeded. These stresses are directly proportional to the radius of curvature of the dome surface. In other words, as the radius of curvature of the dome surface increases, lower air pressures must be provided to maintain the same internal stresses in the fabric. However, as shown later, a minimum amount of air pressure is required such that the fabric is used effectively as a structural form.

Therefore, the radius of curvature of an air form is currently limited to a specific value which is calculated in Chapter 2. Furthermore, a large dome requires additional resistance to external forces such as those induced by snow, wind, and rain. The external forces would likely produce unwanted variations in the air pressure causing the air form to tear and collapse. For these reasons, large concrete domes have not yet been constructed using air-supported forming techniques.

CONTENTS

Several topics are discussed throughout this thesis in an effort to demonstrate that the cable net system is a legitimate solution for the safe construction of large spherical domes. In Chapter 2, the current size limitations of air-supported forms is discussed more in detail, followed by the proposal of two distinct cable net patterns. In addition, an explanation of how the cable net patterns will resolve these size limitations will be presented. In Chapter 3, the radial cable net pattern is outlined. This chapter is divided into several topics which include radius of curvature, air form surface area, cable forces, cable lengths, and stability. The chapter concludes with a discussion of the problems that have surfaced during research associated with this cable net pattern. Next, in Chapter 4, the geodesic cable net pattern is presented. The topics within this chapter parallel those found in Chapter 3 with the addition of two more topics. These topics include a brief summary of

geodesic geometry and typical cable connections. Finally, in Chapter 5, conclusions are drawn and comparisons between the two cable net systems are made.

CHAPTER 2

CABLE NET SOLUTION

As stated in Chapter 1, the air-supported form is only capable of withstanding a certain amount of air pressure before the allowable stresses in the fabric and the welds are exceeded. For any size of dome, the force in the fabric is:

$$N = (p * r_a) / 2 \quad (2.1)$$

where

N = force in the fabric, lb/ft.

p = air pressure used to inflate air form, psf

r_a = radius of curvature of the air form, ft.

The largest thin shell concrete dome constructed with an air-supported form is about 260 ft. in diameter. For this particular dome, the air form thickness is approximately 30 mils which is equivalent to 0.030 in. Since this dome was built as a true hemisphere, the radius of curvature of its air form was exactly half the dome diameter or 130 ft. The air pressure used to inflate this air form was approximately 1.75 in. of water. The equivalent value in psf may be calculated by multiplying the depth of water by its unit weight as follows:

$$\begin{aligned} p &= 1.75 * (1/12) * 62.4 \\ &= 9.1 \text{ psf} \end{aligned}$$

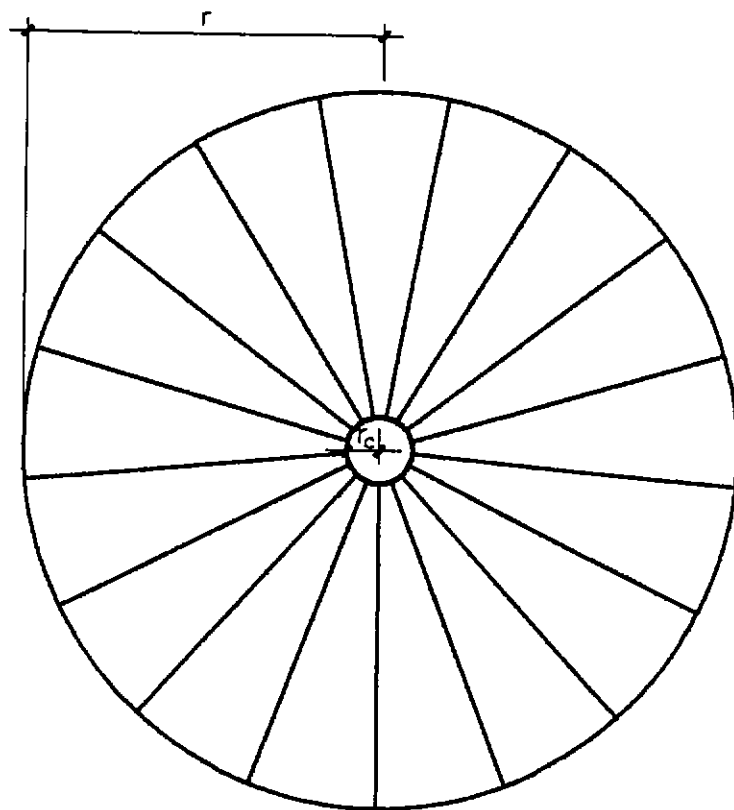
With 9.1 psf air pressure and a 130 ft. radius of curvature, the force in the fabric for the 260 ft. diameter dome is approximately:

$$\begin{aligned} N &= (9.1 * 130) / 2 \\ &= 592 \text{ lb/ft.} \\ &= 49 \text{ lb/in.} \end{aligned}$$

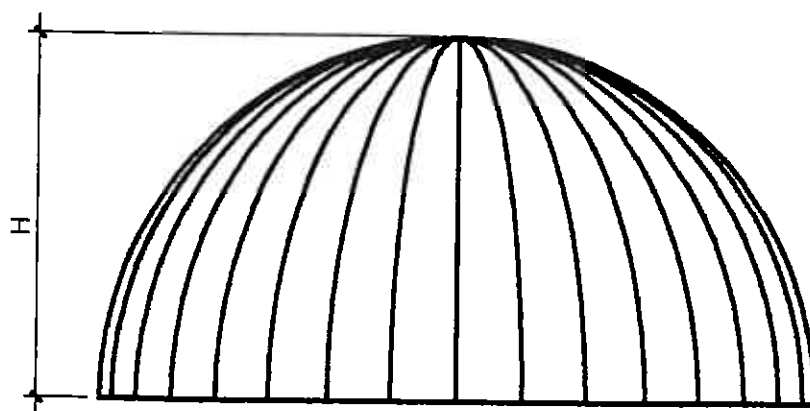
For most air form manufacturers, this force represents an upper limit on the allowable force of the air form fabric. In other words, a 130 ft. radius of curvature is the maximum curvature that an air form manufacturer allows when using a 30 mil fabric. A thicker air form may be more durable but the welds used to manufacture it often control its design. With the use of cable nets, it is possible to decrease the radius of curvature of the air form and still maintain the fabric forces below 49 lb/in. for large spherical domes without increasing the fabric thickness.

CABLE NET PATTERNS

Two distinct cable net patterns have been analyzed for their use with large spherical concrete domes. The first pattern is composed of radial cables that extend from the base of the dome to the apex longitudinally as shown in Figure 2. This pattern includes a small diameter circular cable or 'ring' near the apex. Uniquely, the shape of the finished dome would resemble a half pumpkin. Unfortunately, during the research involving this cable net pattern, specific problems were encountered and will be discussed in detail in Chapter 3. The second cable net pattern involves a geodesic combination of hexagons and pentagons as shown in Figure 3. This pattern is currently more sought after because of its shape,



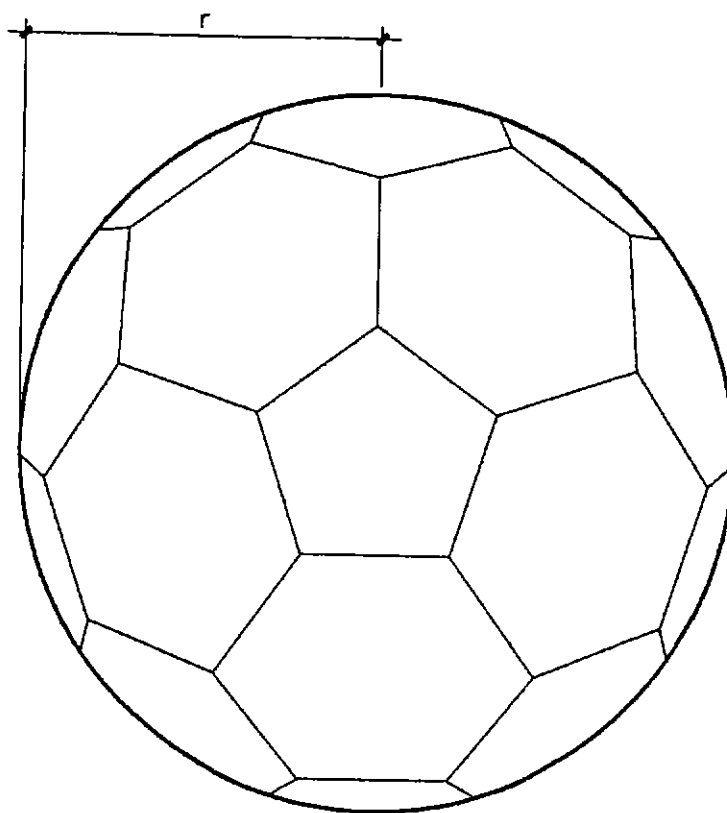
PLAN VIEW



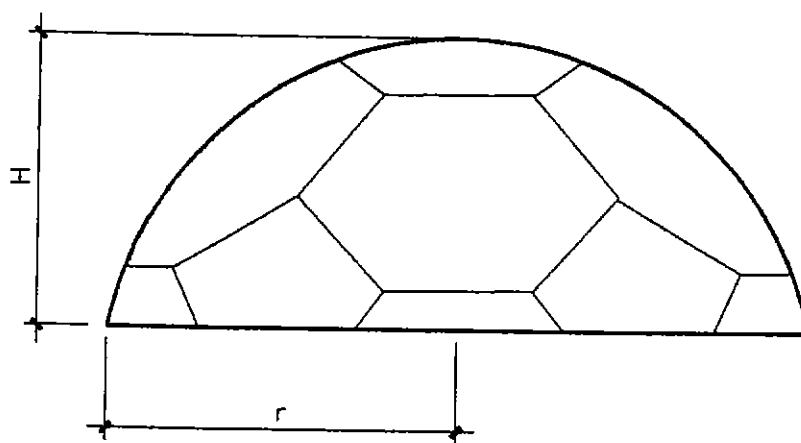
ELEVATION VIEW

RADIAL CABLE NET PATTERN

FIGURE 2



PLAN VIEW



ELEVATION VIEW

GEODESIC CABLE NET PATTERN

FIGURE 3

aesthetics and practicality. ~~The finished dome would resemble aesthetics and practicality.~~ The finished dome would resemble a geodesic pattern or a soccer ball. Throughout this thesis, this cable pattern will be referred to as the geodesic cable net pattern.

It is important to note that all of the cables in each pattern are oriented such that they lay on the surface of a sphere forming only a portion of that sphere (the portion of the sphere will hereafter be called the 'skeleton dome' because the cables act as the 'skeleton' or 'frame' for the structural system). Since all the cables lay on the surface of the same sphere, each cable within a specific cable net pattern has the same constant radius of curvature that is equal to that of the 'skeleton' dome. The radial cable net pattern will be discussed in detail in Chapter 3 while the geodesic cable net pattern will be discussed in Chapter 4.

THE SYSTEM

Now that the cable net patterns have been introduced, the following will briefly describe the properties of the cable net patterns and how they fit in the entire system. The cables are designed to lay over the top of the air form allowing the fabric to 'balloon out' between adjacent cables as the air form is inflated. As the air form 'balloons out', its radius of curvature decreases between the cables. Since the elongation of the air form is minimal, the dimensions and surface area of the fabric that 'balloons out' may be

determined by methods described in Chapters 3 and 4 on air form surface areas.

It is important to note that before the shotcrete is applied to the air form, the cables act as the main structural components for the entire system. The air form does not fully resist the forces that are applied to it, but simply transfers such forces induced by the loadings to the cables. This is quite different than the conventional use of the air-supported form where the air form must withstand all of the forces that are induced by the loadings.

Anchoring the cables to a foundation system such as a continuous footing or a 'ring beam' is the most important part of the cable net system. Connection details will be discussed at the end of Chapter 4. The foundation must be massive enough such that its weight resists the uplift forces described in Chapters 3 and 4. In addition, the connection must be capable of resisting a cable pullout failure in the concrete. In order to maintain a uniform air pressure within the dome, the air form must still be connected to a continuous footing or 'ring beam' as shown in Figure 1. The air form should be sized such that the 'ballooning out' is minimal at the foundation thus preserving the circular nature of the 'ring beam'.

After the air form is fully inflated, the work on the inside of the dome is very similar to the traditional air form method. First, urethane foam is sprayed on the inside surface of the air form providing a means for attaching the

reinforcing steel. Next, shotcrete is applied to the foam in uniform 1/2 in. thick layers after the reinforcing is attached to the foam. Along the cable lines or air form joints, shotcrete is applied such that beams or 'ribs' are formed. These 'ribs' are also built up slowly with 1/2 in. thick layers until the specified thickness is achieved. This shotcrete procedure would commence at the base of the dome and work its way up the air form as the concrete hardens below. The end result will yield a pattern of thickened concrete 'ribs' with sharply curved thin concrete shells between them. The 'ribs' may be as thick as 24 in. or more whereas the concrete shell between the thickened 'ribs' may only be 3 or 4 in. thick. Therefore, these large domes become 'ribbed' domes as a result of the cable nets. After the construction is completed and the concrete has cured, the cables may be removed and reused for a dome of the same dimensions.

CHAPTER 3

RADIAL CABLE NET SYSTEM

The radial cable net system is comprised of several longitudinal cables that extend from the base of the dome to the apex. This chapter will include the essential components that comprise this specific cable net system. These components include the radius of curvature of the air form, the surface areas of the air form, cable forces, cable lengths and overall stability. The problems that have arisen during the research of this cable net pattern which will likely eliminate this cable pattern as a solution for large domes will be discussed at the end of the chapter. It is still convenient, however, to include a Chapter on this cable net pattern so that the problems can be identified and made known to the reader.

RADIUS OF CURVATURE

The radius of curvature of the air form may be considered the most important component of the cable net system. For a 30 mil fabric, the air form radius of curvature must be less than 130 ft. as demonstrated in Chapter 2 such that the fabric forces are not exceeded. It is now important to discuss how the radius of curvature of an air form may be decreased such that 30 mil fabric may still be used for large domes.

Suppose now that a dome which is 500 ft. in diameter and 150 ft. tall is going to be constructed using a 30 mil air-supported form, but without cable nets. The radius of

curvature for the air form as shown on line 8 of Table 1, and illustrated in Figure 4, is:

$$\begin{aligned} r_a &= (r^2 + H^2) / (2 * H) \\ &= (250^2 + 150^2) / (2 * 150) \\ &= 283 \text{ ft.} \end{aligned} \tag{3.1}$$

where

r_a = radius of curvature of the air form, ft.

H = height of the dome, ft.

r = horizontal dome radius at foundation, ft.

From Equation (2.1), the force in the air form fabric, as shown on line 20 of Table 1, would be:

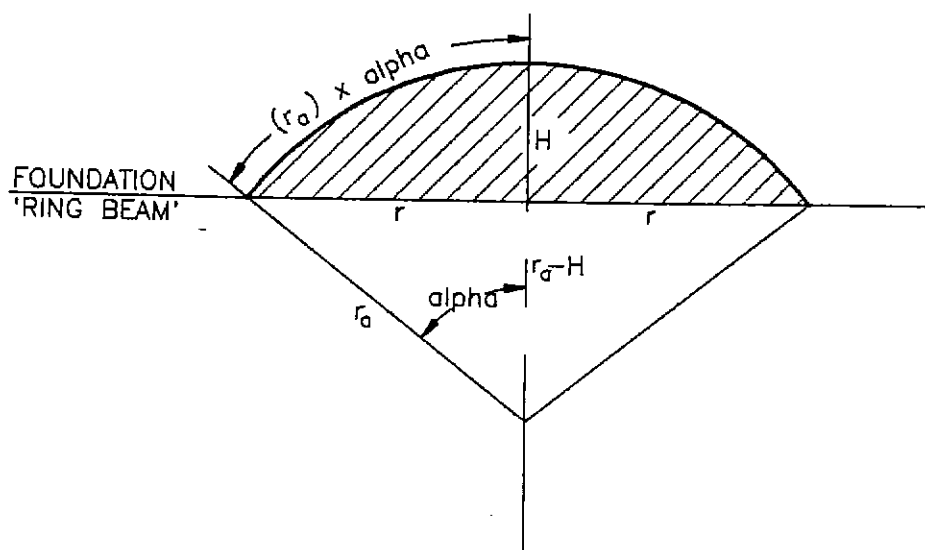
$$\begin{aligned} N &= (9.1 * 283) / 2 \\ &= 1290 \text{ lb/ft.} \\ &= 107 \text{ lb/in.} \end{aligned}$$

Note that with a force of 107 lb/in., the allowable force of the air form would be exceeded by $107 / 49 = 2.2$ times! In fact, the air pressure used to inflate the air form would have to be reduced to 4 psf to maintain a maximum force of 49 lb/in. in the 30 mil air form (Equation (2.1) was used to back calculate p). However, because a 1/2 in. thick layer of concrete weighs just over 6 psf, it is impossible for the air form to support it with only 4 psf of internal pressure. Thus, the construction of large domes using air-supported forming techniques has been inhibited up until the present.

As mentioned before, the cable net system allows the radius of curvature to decrease between adjacent cables. How much 'ballooning out' of the air form is necessary with the

1	Diameter at base of dome (ft) =	300	300	400	500	600	800
2	Number of cables tied to foundation =	16	14	24	22	34	32
3	Dome height, H (ft.) =	100	150	100	150	150	200
4	Rad. of curv. of air form, latitudinally, r(a) (ft) =	50	50	50	50	50	50
5	Thickness of concrete @ modules (in) =	3	3	3	4	4	5
6	Arc dist. between cables along base (ft) =	58.9	67.3	52.4	71.4	55.4	78.5
7	Horizontal angle between cables (degrees) =	22.5	25.7	15.0	16.4	10.6	11.3
8	Radius of curvature of cables, R (ft.) =	162.5	150.0	250.0	283.3	375.0	500.0
9	Angle alpha (degrees) =	67.4	90.0	53.1	61.9	53.1	53.1
10	Diameter of circular cable @ apex (ft) =	11.0	9.0	16.0	14.0	22.0	21.0
11	Vertical rise of module @ apex (ft) =	2.8	2.3	4.0	3.5	5.5	5.3
12	Rad. of curvature of module @ apex, R(c) (ft) =	6.9	5.6	10.0	8.8	13.8	13.1
13	Horiz. thrust in ring with 1/2" conc. layer (lb/ft) =	16.1	13.2	23.4	20.5	32.2	30.8
14	Ring compression on shell @ circ. rib, C (lb) =	88.6	59.3	187.5	143.7	354.5	323.0
15	Length of each cable required, L (ft/cable) =	184.7	230.4	222.6	298.1	335.0	451.5
16	Total uplift force, U (kips) =	643	643	1144	1787	2573	4574
17	Uplift force per tributary width of fdn., u (kips) =	40.2	45.9	47.6	81.2	75.7	142.9
18	Force in cables, F (kips) =	43.6	45.9	59.6	92.0	94.6	178.7
19	Surface area of 'skeleton' dome, A (ft ²) =	102102	141372	157080	267035	353429	628319
20	Fabric force in 'skeleton' dome (lb/ft) =	61.6	56.9	94.8	107.4	142.2	189.6
21	Fabric force in module, N (lb/ft) =	19.0	19.0	19.0	19.0	19.0	19.0
22	Strand diameter (in) =	1.000	0.875	1.000	1.250	1.250	1.625
23	Allowable tensile strength of strand (kips) =	61	46	61	96	96	162
24	Weight of strand (lb/ft) =	2.10	1.61	2.10	3.28	3.28	5.55
25	Total length of strand (ft.) =	2990	3254	5391	6603	11459	14513
26	Total weight of strand (kips) =	6.2	5.2	11.2	21.5	37.4	80.2
27	Weight of strand (psf) =	0.06	0.04	0.07	0.08	0.11	0.13
28	Cost of strand (includes labor/connections) =	\$14,951	\$16,270	\$26,957	\$33,013	\$57,293	\$72,566

RADIAL CABLE NET CALCULATIONS
TABLE 1



RADIUS OF CURVATURE

FIGURE 4

radial cable net system such that the fabric forces are maintained below 49 lb/in.? With some guidance, the answer to this question is left up to the air-form manufacturer because it has an infinite number of correct solutions.

Theoretically, any radius of curvature less than 130 ft. should accomplish the task of maintaining the fabric forces below 49 lb/in. in a 30 mil fabric. It is recommended, however, that a factor of safety of at least 2 be utilized in making this decision. In addition, a rule of thumb states that the dome rise to span should be limited to a ratio of 1:8 because of shell stability considerations such as buckling or 'caving in'. For large domes, this ratio has been known to increase to 1:4. This rule of thumb could also be applied to the portion of the air form that 'balloons out' between cables such that the span is the distance between adjacent cables and the rise is the distance that the air form 'balloons out'. For the 500 by 150 ft. example dome, a radius of curvature of 50 ft. has been chosen for calculation purposes as shown on line 4 of Table 1. Note that the 50 ft. represents the radius of curvature of the air form and not that of the cables. With this new radius of curvature between cables, the force in the air form fabric, as shown on line 21 of Table 1, is reduced to:

$$\begin{aligned} N &= (9.1 * 50) / 2 \\ &= 228 \text{ lb/ft.} \\ &= 19 \text{ lb/in.} \end{aligned}$$

In Chapter 2, it was mentioned that the radial cable net included a short circular cable located at the apex of the dome as shown in Figure 2. The addition of this cable serves a twofold purpose. First, it aids in increasing the stability of the dome as discussed in the stability section below. Second, it helps to maintain a sharp radius of curvature at the apex to maintain the air form forces below the allowable values. This cable is designed so that the fabric which is circumscribed by the cable will 'balloon out' forming a 'small' dome (hereafter called module) at the apex of the dome. The air form radius of curvature for this module may be calculated using Equation (3.1) and substituting the height of the module for the height of the entire dome and replacing the horizontal dome radius at the foundation, r , with r_c , the horizontal radius of the circular cable at the dome apex. Using Equation (3.1), the radius of curvature of the module at the apex of the 500 by 150 ft. dome is 8.8 ft. as shown on line 12 of Table 1. The diameter of the circular cable is 14 ft. as shown on line 10 of Table 1. It was calculated in accordance with criteria established in the section on cable lengths below. The height of the module, as shown on line 11 of Table 1, was chosen as 3.5 ft. in accordance with the 1:4 rule of thumb discussed above.

AIR FORM SURFACE AREA

The surface area of the air form that 'balloons out' is very difficult to determine. Not only does the air form

contain two distinct curvature radii, but the curvature in one direction is variable. In the radial or longitudinal direction, the radius of curvature of the air form is approximately equal to that of the radial cables. Latitudinally, the air form possesses a distinct radius of curvature that is much sharper. It is this radius of curvature that is discussed above that must be carefully selected such that the allowable forces in the air form are not exceeded.

Given the two distinct curvature radii, it is difficult to determine just what the surface area of the air form must be. Therefore, without drawing a three dimensional model on a computer system to determine the surface area of these dual curved air forms, only a rough approximation is available. Fortunately, most air-supported form manufacturers have the capability of this type of computer system.

For an approximation of the surface area, it is recommended to start with the surface area of the 'skeleton' dome which does not include the extra surface area obtained through the 'ballooning out' of the fabric. Equation (3.8), introduced in the stability section of this chapter, may be used to calculate such surface area. The surface area of the air form that 'balloons out' between cables could be calculated by multiplying the 'skeleton' surface area by a calculated factor. Whether this factor is assumed or a three dimensional computer model is generated to obtain the surface

area of the proposed style of domes, the actual surface area calculation is left up to the air form manufacturer.

CABLE FORCES

The determining of the cable forces is another essential component of the cable net system. As shown on line 22 of Table 1, different cable diameters have been selected for each dome size depending on the forces in the radial cables. Note that all of the calculations are based on an air pressure inflation of 1.75 in. of water.

All of the following example calculations will be based on a concrete dome with the following dimensions and properties:

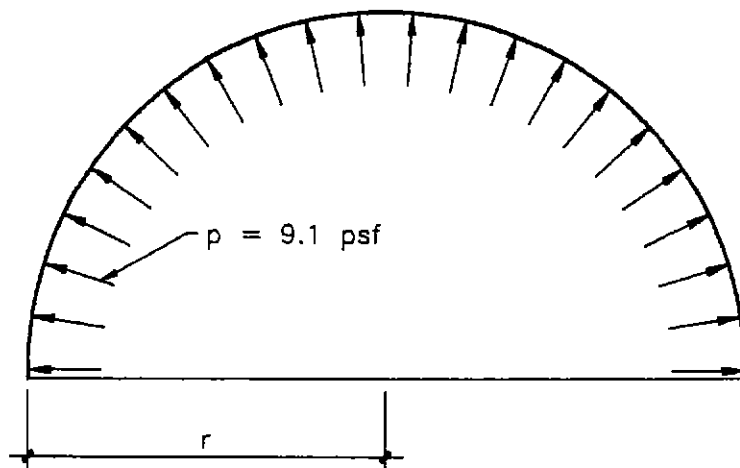
Dome diameter: 500 ft.

Dome Height: 150 ft.

Cable diameter: 1.25 in.

The results of the equations that follow may be compared to the values shown in Table 1 for a dome of the same dimensions and properties. Since significant figures are utilized in the following calculations, the values in the text may not exactly coincide with those found in the tables.

Before determining the forces in the cables, the total uplift force on the air form must be calculated. Said uplift force is the sum of all the vertical components of the air pressure that act perpendicular to the air form surface as shown in Figure 5. Therefore, the uplift force is a function of both the air pressure within the air form and its



AIR PRESSURE WITHIN AIR FORM

FIGURE 5

vertically projected surface area, i.e. floor area. For a 500 ft. diameter spherical dome, regardless of its height, the total uplift force on the air form, as shown on line 16 of Table 1, is:

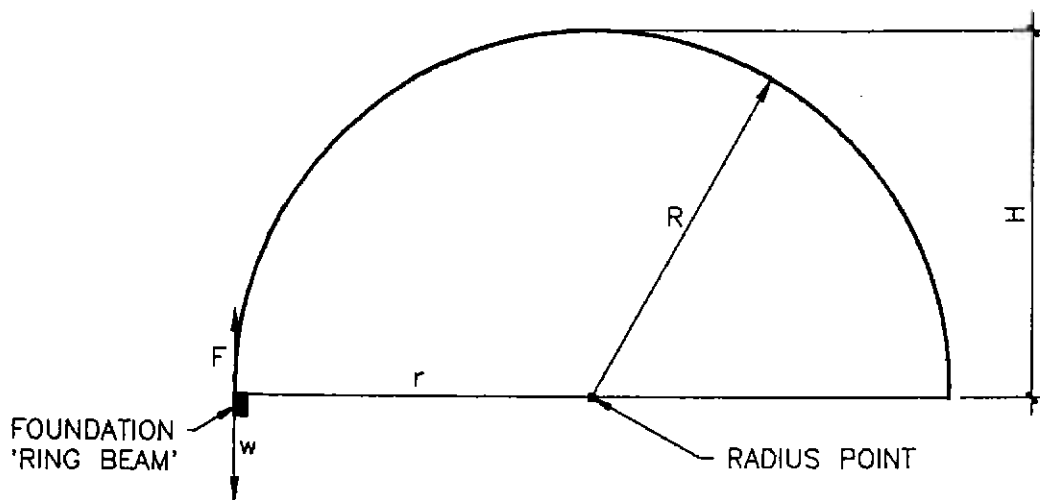
$$\begin{aligned} U &= p * \pi * r^2 & (3.3) \\ &= \{ 9.1 * 3.14 * (250)^2 \} * (1/1000) \\ &= 1790 \text{ kips} \end{aligned}$$

where

- U = Total uplift force on the air form, kips.
- p = air pressure used to inflate air form, psf.
- r = horizontal dome radius at foundation, ft.

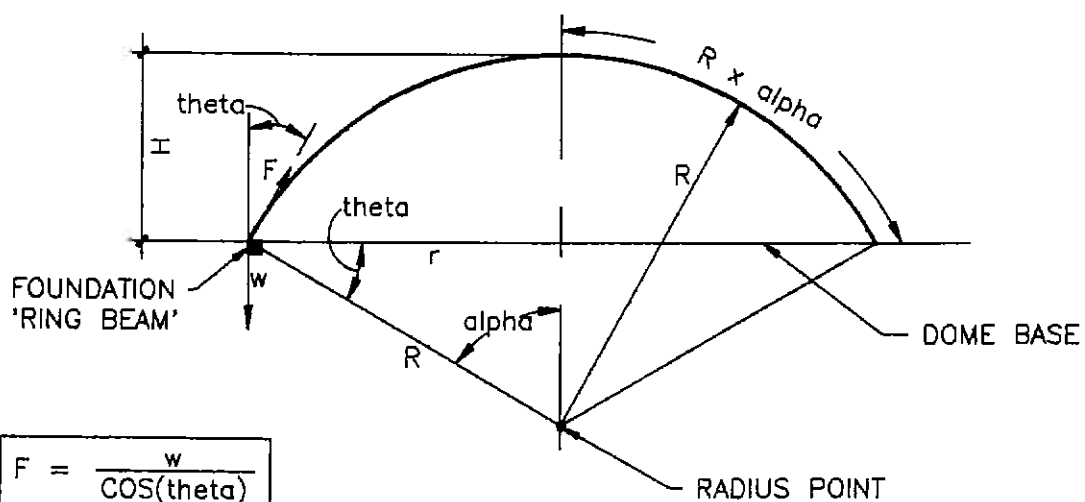
As explained above, the total uplift force acts vertically upward. By Newton's 3rd law, there must be an equal and opposite vertically downward force opposing the vertically upward one. The downward force is primarily the weight of the foundation system. Thus, the total weight of the foundation system, W , should provide the resistance to the total uplift force, U . If the air form is inflated uniformly, then the foundation system or 'ring beam' should theoretically resist the total uplift force uniformly around its perimeter.

As shown in Figure 6, the weight of a tributary length of the 'ring beam', w , between two adjacent cables must be at least equal to the upward force or tensile force, F , of its corresponding cable for a hemispherical dome. In the same manner, the cable force, F , must be equal to the uplift force associated with a tributary length of 'ring beam', u . For



$$\begin{aligned} F &= w \\ &= u \end{aligned}$$

HEMISPHERICAL DOME



$$\begin{aligned} F &= \frac{w}{\cos(\theta)} \\ &= \frac{u}{\cos(\theta)} \end{aligned}$$

PARTIAL HEMISPHERICAL DOME

CABLE FORCE VECTORS

FIGURE 6

example, on line 6 of Table 1, it shows that the arc distance between cables along the perimeter base of the 500 by 150 ft. dome is 71.4 ft. This arc distance represents the tributary length of the 'ring beam', w , mentioned above. If the 'ring beam' were 1.5 ft. wide and 2 ft. deep, then the weight of the tributary length of 'ring beam', w , would be $1.5 * 2 * 30.8 * 0.150 = 14$ kips, which must be at least equal to the tensile force, F , in the cable. Therefore, for a hemispherical dome, the tensile force in each cable may be determined by dividing the total uplift force on the air form by the number of cables in the pattern. Likewise, the number of cables required may be calculated by dividing the total uplift force by the allowable strength of the cables. Setting the cable force, F , equal to the weight of the corresponding tributary length of 'ring beam', w , is done to illustrate a concept. Note that w should be somewhat greater than F to ensure that the cable force never exceeds the foundation weight.

Galvanized multiple-wire strands have been recommended by air form manufacturers for their use in the proposed cable nets because of their limited elongation. Table 2 provides some of the structural properties of the multiple-wire strands that are accepted by the ASTM A586 Specification. As noted in the table, the minimum breaking strength of a 1.25 in. diameter (19 wire) strand is 96 tons or 192 kips. It is recommended that a factor of safety of 2 be used when utilizing these strands in cable nets to account for variations in air pressure on the air form. As shown in row

*Minimum breaking strengths are based on furnishing Class B or C zinc coating weights on the outside wires, with Class A on the inside wires. The heavier Class B and C zinc coatings reduce the steel metallic area, which accounts for the slightly lower strengths.

Minimum moduli of elasticity of the above strands, when prestretched, are as follows:

1/2-in. to 2 1/16-in. diam 24,000,000 psi
2 5/8-in. and larger 23,000,000 psi

Moduli are based on Class "A" coating; for heavier coatings, reduce modulus approximately 1,000,000 psi.

Diam in.	Minimum Breaking Strength, tons			Metallic Area, approx sq in.	Weight per ft., approx lb
	Class A Coating	*Class B Coating	*Class C Coating		
1/2	15	14.5	14.2	.150	.52
5/16	19	18.4	18.0	.190	.66
3/8	24	23.3	22.8	.234	.82
7/16	29	28.1	27.5	.284	.99
1/2	34	33.0	32.3	.338	1.18
13/16	40	38.8	38.0	.396	1.39
3/8	46	44.6	43.7	.459	1.61
15/16	54	52.4	51.3	.527	1.85
1	61	59.2	57.9	.600	2.10
1 1/16	69	66.9	65.5	.677	2.37
1 1/8	78	75.7	74.1	.759	2.66
1 1/4	86	83.4	81.7	.846	2.96
1 1/2	96	94.1	92.2	.938	3.28
1 5/8	106	104	102	1.03	3.62
1 3/4	116	114	111	1.13	3.97
1 7/8	126	123	121	1.24	4.34
2	138	135	132	1.35	4.73
2 1/16	150	147	144	1.47	5.13
2 1/8	162	159	155	1.59	5.55
2 1/4	176	172	169	1.71	5.98
2 3/8	188	184	180	1.84	6.43
2 1/2	202	198	194	1.97	6.90
2 5/8	216	212	207	2.11	7.39
2 3/4	230	226	221	2.25	7.89
2 7/8	245	241	238	2.40	8.40
3	261	257	253	2.55	8.94
3 1/16	277	273	269	2.71	9.49
3 1/8	293	289	284	2.87	10.1
3 1/4	310	305	301	3.04	10.6
3 1/2	327	322	317	3.21	11.2
3 3/8	344	339	334	3.38	11.8
3 1/4	360	355	349	3.57	12.5
3 1/2	376	370	365	3.75	13.1
3 5/8	392	386	380	3.94	13.8
3 3/4	417	411	404	4.13	14.5
3 7/8	432	425	419	4.33	15.2
4	452	445	438	4.54	15.9
4 1/16	494	486	479	4.96	17.4
4 1/8	538	530	522	5.40	18.9
4 1/4	584	575	566	5.86	20.5
4 1/2	625	616	606	6.34	22.2
4 3/8	673	663	653	6.83	23.9
4 1/4	724	713	702	7.35	25.7
4 1/2	768	756	745	7.88	27.6
4 3/4	822	810	797	8.43	29.5
4 1/2	878	865	852	9.00	31.5
4 3/4	925	911	897	9.60	33.6

MULTIPLE-WIRE STRAND PROPERTIES

TABLE 2

23 of Table 1, the tensile strength of the 1.25 in. diameter (19 wire) strand which is used for these cable force calculations is $192 / 2 = 96$ kips. With an uplift force of 1790 kips from Equation (3.3), the number of cables required is $1790 / 96 = 19$ cables. Notice that this does not correspond to the requirement of 22 cables as shown on line 2 of Table 1. This is because a 500 by 150 ft. dome is not a true hemisphere.

For a hemispherical dome, the upward force on the foundation (the tensile force in the cable) is vertically upward as shown in Figure 6. For domes that are partial hemispheres, some trigonometry must be used. In order to determine the true tension in the cable, one must determine the angle at which the cable meets the foundation from vertical. The actual cable force, F , can then be computed by dividing the uplift force, u , associated with each tributary length of 'ring beam' by the cosine of the angle θ as shown in Figure 6. Likewise, the number of cables required may be determined by dividing the cable force, F , attributed to each tributary length of 'ring beam' by the product of the cosine of the angle θ and the allowable tensile strength of the cable. The required angle, θ , for the 500 x 150 ft. dome may be determined from the radius of curvature of the cables which radius of curvature is equal to that of the air form calculated as 283 ft. from Equation (3.1).

Both the horizontal radius of the dome and the radius of curvature of the cables are now known. By trigonometry, the

sine of the angle alpha, as shown in Figure 6, is equal to the horizontal dome radius, r, divided by the radius of curvature of the cables, R. Thus the angle alpha, as shown on line 9 on Table 1, is:

$$\begin{aligned}
 \alpha &= \text{SIN}^{-1}(r / R) & (3.4) \\
 &= \text{SIN}^{-1}(250 / 283) \\
 &= 61.9 \text{ degrees} \\
 &= 1.08 \text{ radians}
 \end{aligned}$$

By geometry, the complement of the angle alpha is theta, as shown in Figure 6 and is $90 - 61.9 = 28.1$ degrees. The number of cables required, as shown on line 2 of Table 1, is therefore $19 / \text{COS}(28.1) = 22$ cables. The actual force in the cable, as shown on line 18 of Table 1, is $1790 / (22 * \text{COS}(28.1)) = 92$ kips.

CABLE LENGTHS

The individual length of each cable is also an essential component of the entire system. The cable lengths must be exact such that they are stiff and fit firm to the air form. Using arc length principles, these lengths may be calculated precisely by using the angle alpha as introduced above. The length of each cable in the example dome will be:

$$\begin{aligned}
 L &= R * \alpha & (3.5) \\
 &= 283 * 1.08 \\
 &= 305 \text{ ft.}
 \end{aligned}$$

where R is in ft. and alpha is in radians.

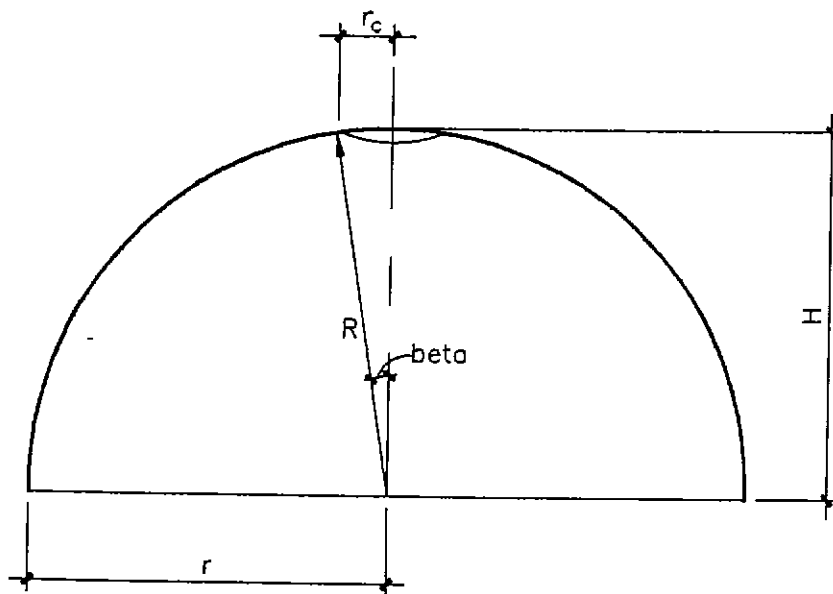
Notice that this does not correspond to the requirement of 298.1 ft. of cable as shown in line 15 of Table 1. Remember that this cable pattern includes a 'small' diameter circular cable or 'ring' near the apex. The diameter of this 'ring' is large enough to facilitate the connection of all the cables. These connections are covered at the end of Chapter 4. The diameter of the 'ring' is such that at least 2 ft. of circular cable exists between each adjacent incoming radial cable. This calculation also complies with arc length principles. For example, as shown on line 7 in Table 1, the horizontal angle between each cable at the apex of the dome (or at any height) is 16.4 degrees or 0.29 radians. This is found by dividing 360 degrees by 22, the number of cables. In order to obtain an arc length of 2 ft., the radius of the circular cable must be at least $2 / (0.29) = 7$ ft which is equivalent to a 14 ft. diameter circle as shown on line 10 of Table 1.

The circular cable will reduce the length of each radial cable by a few ft. This reduction can also be determined using arc lengths and trigonometry. The angle beta as shown in Figure 7, is:

$$\begin{aligned}
 \text{beta} &= \text{SIN}^{-1}(r_c / R) & (3.6) \\
 &= \text{SIN}^{-1}(7 / 283) \\
 &= 1.4 \text{ degrees} \\
 &= 0.03 \text{ radians}
 \end{aligned}$$

where

R = Radius of curvature of the cable net, ft.



SMALL CIRCULAR CABLE ● APEX

FIGURE 7

r_c = Horizontal radius of circular cable at apex, ft.

The reduced length of the radial cable is:

$$\begin{aligned} l &= R * \text{beta} \\ &= 283 * 0.03 \\ &= 7 \text{ ft.} \end{aligned} \tag{3.7}$$

where R is in ft. and beta is in radians.

The radial cable length is therefore $305 - 7 = 298$ ft. and the total length of cable required for this dome is $(298 * 22) + (\pi * 14) = 6,600$ ft. as shown on lines 15 and 25 of Table 1, respectively.

According to Table 2, a 1.25 in. diameter cable weighs 3.28 lb/ft. Therefore, the total weight of the cable is $6,600 * 3.28 * (1 / 1000) = 22$ kips as shown on line 26 of Table 1. At approximately \$5.00 per ft. of installed cable, the cable and required connections will add approximately $6,600 * 5 = \$33,000$ to the cost of project as shown on line 28 of Table 1. In comparison to a concrete dome which is conventionally formed without air-supported forms and cable nets, this is a reasonable cost as demonstrated in Chapter 5.

STABILITY

The stability of the air-supported form has always been a major concern. These concerns involve the possible collapse of the concrete, the steel, or the air form. These failures could be caused by an insufficient amount or variation of air pressure within the air form or could be induced by a non-uniform concrete application. For the purposes of this

section on stability, only the collapse of the concrete will be discussed.

As mentioned previously, reinforced concrete weighs just over 6 psf while an inflation of 1.75 in. of water results in an outward pressure of 9.1 psf on the air form. The total weight of cable for the 500 by 150 ft. dome was calculated at 22 kips. This weight can be resolved into psf by determining the surface area over which the cables cover. It is conservative to use the surface area of the 'skeleton dome' that is formed by the cable net which does not include the extra surface area obtained by the 'ballooning out' of the air form between cables. The surface area, where the angle alpha is shown on Figure 6, is:

$$\begin{aligned}
 A &= 2 * \pi * R^2 * (1 - \cos(\alpha)) & (3.8) \\
 &= 2 * 3.14 * 283^2 * (1 - \cos(1.08)) \\
 &= 267,000 \text{ sq ft.}
 \end{aligned}$$

where

A = Surface area of the skeleton dome, ft².

R = Radius of curvature of the cable net, ft.

Note that this surface area confirms the value tabulated on line 19 of Table 1. The weight of the cables is thus 22,000 / 267,000 = 0.08 psf as shown on line 27 of Table 1. This weight is so insignificant that the air form will not detect that the cable is there!

Because of the shape of the dome, there will be stress concentrations in the concrete at the joints (valley's) where the cables lie. The concrete at these joints must be

thickened to form 'ribs' such that these joints act as beams that are capable of transferring loads to the foundation. The sizes of these 'ribs' will have to be determined by a finite element analysis as mentioned in Chapter 1. However, it is possible to calculate the axial compression in the beams induced by the 'small' circular dome at the apex of the air form.

In the cable length section of this chapter, the diameter of the circular cable at the apex of a 500 by 150 ft. dome was calculated as 14 ft. The length of this cable or the perimeter of the circle formed by it is $3.14 * 14 = 44$ ft.

Suppose now that the shotcrete is being applied to the air form and the workers have finally reached the apex and are in the process of applying a 1/2 in. thick layer of concrete to the module at the apex. Using Equation (3.4) where r is replaced with r_c , the horizontal radius of the circular cable at the dome apex and R is replaced by R_c , the radius of curvature of the module (8.8 ft.), the new angle alpha for this module is $\text{SIN}^{-1}(7 / 8.8) = 53$ degrees. Applying Equation (3.8) with alpha equal to 53 degrees, the surface area of the air form within the module is 194 ft². With a 1/2 in. thick shell between 'ribs', this amounts to approximately 1,200 lb. of concrete if it is applied uniformly to the air form.

At this point, a thickened concrete rib should be built up at the joint formed by the air form and the circular cable. The weight of the concrete at the apex of the dome could be resolved into a force which is applied along the length of the

rib, which would be $1,200 / (3.14 * 14) = 27 \text{ lb/ft}$. Since this force is analogous to a uniform load around a skylight opening for conventional concrete domes, the ring compression induced into the edge of the shell may be calculated using conventional techniques. On page 115 of David P. Billington's "Thin Shell Structures", second edition, the ring compression, as shown on line 14 of Table 1, is:

$$\begin{aligned} C &= R_c * P * \cos(\alpha) \\ &= 8.8 * 27 * \cos(53) \\ &= 145 \text{ lb.} \end{aligned} \tag{3.9}$$

where

C = Ring compression on shell at circular 'rib', lb.

P = Load along perimeter of circular 'rib', lb/ft.

R_c = Radius of curv. of the module at apex, ft.

This compression force is then distributed among all of the radial 'ribs' making it so small that it is very insignificant.

In short, if the shotcrete is applied uniformly in 1/2" maximum layers, and if the air form that 'balloons out' maintains a minimum ratio of rise to span of 1:4, and the air pressure is sufficient and continuously monitored, then the stability of the dome should be sound.

PROBLEMS AND LIMITATIONS

As mentioned briefly in Chapter 2, specific problems with this cable net pattern have been encountered during the research. As shown in Figure 5, the air pressure within the

air form acts perpendicularly to the air form surface. Thus the pressure that acts on the air form must be transferred to the nearest cable. The total force per linear ft. that acts along each cable may be determined by multiplying the pressure, p , by the tributary width of the air form associated with a particular cable at a specific location. As shown in Figure 2, the tributary width of the air form is much greater at the base of the dome than at the apex. Therefore, a greater force will be exerted on the lower portions of the cable causing the desired spherical shape to assume an elliptical one. This would tend to lower the apex elevation and increase the radius of curvature at the apex. In fact, as the pressure within the air form is increased, the height of the apex will likely decrease. If this phenomena occurs, then the purpose of this cable net system is defeated because it would worsen the construction problems as a result of the increase in air form radius of curvature.

CHAPTER 4

GEODESIC CABLE NET SYSTEM

In comparison with the radial cable net, the geodesic pattern is a more practical choice for some air-supported form manufacturers. Since the cables in the geodesic cable net are more uniformly spread out over the air form surface, the forces are more evenly distributed between the cables. The geodesic cable pattern is also practical because the general public enjoys the uniqueness of geodesic shapes. As shown in Figure 3, this cable pattern involves alternating rows of cables that form hexagonal and pentagonal shapes. Note that a pentagon is centered about the apex of the dome followed by a row of hexagons, and so forth. This chapter will include the primary elements required to illustrate that this cable net system can be effectively utilized for large spherical domes. As with the radial cable system, these elements include the radius of curvature of the air form, the surface area of the air form, cable forces, cable lengths and overall stability. A brief introduction to geodesic geometry will also be given. This chapter will conclude with a discussion on cable connections.

RADIUS OF CURVATURE

Unlike the radial cable net system, the radius of curvature of the geodesic pattern is not so complex. Each hexagon and pentagon (hereafter called modules) is associated

with the same radius of curvature in both the latitudinal and longitudinal directions. As stated in Chapter 2, the air form radius of curvature must be less than 130 ft. in order to maintain the forces in the fabric and in the welds below the allowable value of 49 lb/in. for a 30 mil fabric.

It is now convenient to again consider the 500 ft. diameter by 150 ft. tall dome such that comparisons between each cable pattern system may be made. Note that on line 3 of Table 3, the 500 ft. dome was actually analyzed with a height of 154.5 ft., which height is determined by geodesics as explained later in the section on geodesic cable lengths. From Equation (3.1), it was found that the radius of curvature of the dome's air form without the use of cable nets is 283 ft. Note that if a cable net system were used, regardless of which type, the radius of curvature of each cable would also be 283 ft. Due to the height adjustment, the actual radius of curvature of the cables for the 500 ft. diameter geodesic dome is 279.5 ft. as shown on line 5 of Table 3. In Chapter 3, it was shown that without using cable nets, the force in the air form fabric for such a radius of curvature would be about 107 lb/in. which is well above the allowable force of 49 lb/in. Again, this demonstrates that the radius of curvature of the air form must be decreased for large domes. As before, the decrease in radius of curvature comes about by allowing the air form to 'balloon out' between the cables, thus forming modules. The geodesic cable pattern must now be sized such that the radius of curvature may be defined.

Uplift force assumes a full balloon load at 1.75" water (9.1 psf) Inflation:

1	Diameter at base of Dome (ft.) =	300	300	400	500	600	800
2	Number of cables tied to foundation =	20	20	30	30	40	40
3	Dome height, H (ft.) =	92.71	150	123.61	154.51	159.85	213.13
4	Icosaheron frequency utilized, f =	6	6	9	9	15	15
5	Radius of curvature of cables, R (ft.) =	167.7	150.0	223.6	279.5	361.4	481.9
6	Angle alpha, cables (degrees) =	63.4	90.0	63.4	83.4	56.1	56.1
7	Total uplift force, U (kips) =	643	643	1144	1787	2573	4574
8	Uplift force per tributary width of fdn., u (kips) =	32.2	32.2	38.1	58.6	64.3	114.4
9	Circular approx. @ base of module (ft) =	64.7	64.7	58.2	72.8	52.7	70.3
10	Height of module (ft) =	16.2	16.2	14.5	18.2	13.2	17.6
11	Angle alpha for module (deg.) =	53.1	53.1	53.1	53.1	53.1	53.1
12	Radius of curvature of module, r(a) (ft) =	40.5	40.5	36.4	45.5	33.0	43.9
13	Air form force in module, N (lb/in) =	15.3	15.3	13.8	17.3	12.5	16.7
14	Approx. surface area of module (ft^2) =	3402.6	3402.6	2748.9	4302.7	2258.6	4015.3
15	Floor area under dome (ft^2) =	70885.8	70885.8	125663.7	198349.5	282743.3	502654.8
16	Surface area of 'skeleton' dome, A (ft^2) =	97685.4	141371.7	173663.0	271348.4	363016.2	645362.1
17	Force in cables, F (kips) =	50.4	37.0	60.5	94.5	87.0	154.7
18	Guy strand diameter (in) =	1.000	0.875	1.000	1.250	1.250	1.625
19	Allowable tensile strength of strand (kips) =	61	46	61	96	96	162
20	Weight of strand (lb/ft) =	2.10	1.61	2.10	3.28	3.28	5.55
21	Total length of strand (ft.) =	2731.7	5268.1	5904.0	7396.1	6723.4	7300.5
22	Total weight of strand (kips) =	5.7	8.5	12.4	24.3	22.1	40.5
23	Weight of strand (psf) =	0.06	0.06	0.07	0.09	0.06	0.06
24	Cost of strand (includes labor/connections) =	\$13,658	\$26,340	\$29,520	\$36,981	\$33,617	\$36,503

GEODESIC CABLE NET CALCULATIONS

TABLE 3

Recently, recommendations were made by an air form manufacturer that the modules be sized such that they are circumscribed by a circle with an approximate diameter of 60 to 70 ft. in order to provide sufficient area for the air form to 'balloon out'. Therefore, all the vertices of each module (i.e. cable junctions) touch the perimeter of the circle. For the 500 ft. dome, the air form that 'balloons out' within each module is modeled as a smaller dome with an equivalent circular base diameter of 72.8 ft., as shown on line 9 of Table 3. The calculation of this equivalent diameter will be shown below in the cable lengths section of this chapter. If the 1:4 ratio of dome rise to span is utilized, then the air form would theoretically 'balloon out' approximately 18.2 ft. perpendicular to the plane of the 72.8 ft. diameter circular base, as shown on line 10 of Table 3. Using Equation (3.1) and the circular base approximation, the radius of curvature of each module, as shown on line 12 of Table 3, is approximately:

$$\begin{aligned} r_s &= (36.4^2 + 18.2^2) / (2 * 18.2) \\ &= 45.5 \text{ ft.} \end{aligned}$$

With this radius of curvature and using Equation (2.1), the force in the air form fabric within each module, as shown on line 13 of Table 5, is approximately:

$$\begin{aligned} N &= (9.1 * 45.5) / 2 \\ &= 207 \text{ lb/ft.} \\ &= 17.3 \text{ lb/in.} \end{aligned}$$

Notice that both the radius of curvature and the force in the fabric within each module are both well below the allowables of 130 ft. and 49 lb/in., respectively.

AIR FORM SURFACE AREA

Unlike the radial cable net system, the surface area of each module in the geodesic cable net system is easier to estimate because of its constant radius of curvature. As far as the procedure is concerned, the surface area within each hexagon and pentagon will first be determined. This surface area is then multiplied by the sum of hexagons and pentagons in the pattern for a particular dome in order to obtain an approximate total surface area of air form required.

The surface area of each module of the 500 ft. dome may be approximated by modeling a dome with a circular base diameter of 72.8 ft. With the use of Equation (3.4), the angle alpha for a 72.8 ft. diameter by 18.2 ft. tall dome (module), as shown on line 11 of Table 3 and in Figure 6, is approximately:

$$\begin{aligned}\alpha &= \text{SIN}^{-1}(36.4 / 45.5) \\ &= 53.1 \text{ degrees} \\ &= 0.93 \text{ radians}\end{aligned}$$

Now using Equation (3.8), the surface area of this module, as shown on line 14 of Table 3, is approximately:

$$\begin{aligned}A &= 2 * 3.14 * 45.5^2 * (1 - \text{COS}(53.1)) \\ &= 5,200 \text{ ft}^2.\end{aligned}$$

Because of the circular base approximation of each module, there is an extra portion of the total surface area included that is bounded by the circle and each polygon. In plane geometry, the area of a particular hexagon is always 20.9 percent greater than the area of the circle that circumscribes it. Therefore, the surface area of each module is closer to $5,200 / 1.209 = 4300 \text{ ft}^2$. Given the overall dimensions of the geodesic pattern, the surface area of the modules should be determined more precisely by using a three dimensional computer model. Once the total surface area of each module is determined, then it is simply multiplied by the total number of modules which will hereafter be determined in the cable length section of this chapter.

INTRODUCTION TO CABLE FORCES

The calculation of the cable forces for the geodesic pattern is more complicated than the radial cable net system. In order to compare the two proposed cable net systems, the same cable diameters have been selected for their use in similarly sized domes as shown in Table 3. Note that all of the calculations are also based on an air form inflation of 1.75 in. of water pressure.

As with the radial cable system, with the exception of the height adjustment, the example calculations will be based on a concrete dome with the following dimensions and properties:

Dome diameter: 500 ft.
Dome Height: 154.5 ft.
Cable diameter: 1.25 in.

The results of the equations that follow may be compared to the values shown in Table 3 for a dome of the same dimensions and properties. Since significant figures are utilized in the following calculations, the values in the text may not exactly coincide with those found in the tables.

The total uplift force on the air form for this dome is identical to that of the dome analyzed in Chapter 3 since the uplift force is a function of the inflation pressure and the base area of the dome. From Equation (3.3), this force was calculated at 1790 kips, also shown on line 7 of Table 3. As before with the radial cable net system, the tensile force in each cable may be determined by dividing the total uplift force on the air form by the number of cables that connect to the foundation. The number of cables that are fastened to the foundation depends on the geodesic geometry of the cable net. As shown on line 2 of Table 3, the 500 ft. dome contains 30 cables which are fastened to the foundation. Therefore, the uplift force, u , per tributary width of foundation is $1790 / 30 = 60$ kips, as shown on line 8 of Table 3.

In a geodesic cable net, there are two angles that must be considered when calculating the tensile force in the cable. Suppose that the cable is located on the North side of the air form. Looking at the East elevation of the dome, the angle at which the cable meets the foundation from vertical represents

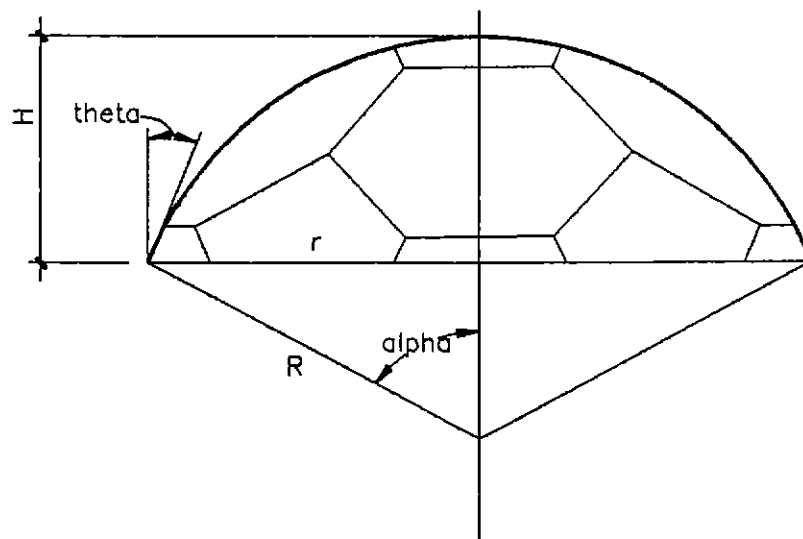
the angle theta as discussed in Chapter 3 and shown in Figure 8. Using Equation (3.4), the angle alpha for this dome, as shown on line 6 of Table 3, is approximately:

$$\begin{aligned}\alpha &= \sin^{-1}(250 / 279.5) \\ &= 63.4 \text{ degrees} \\ &= 1.1 \text{ radians}\end{aligned}$$

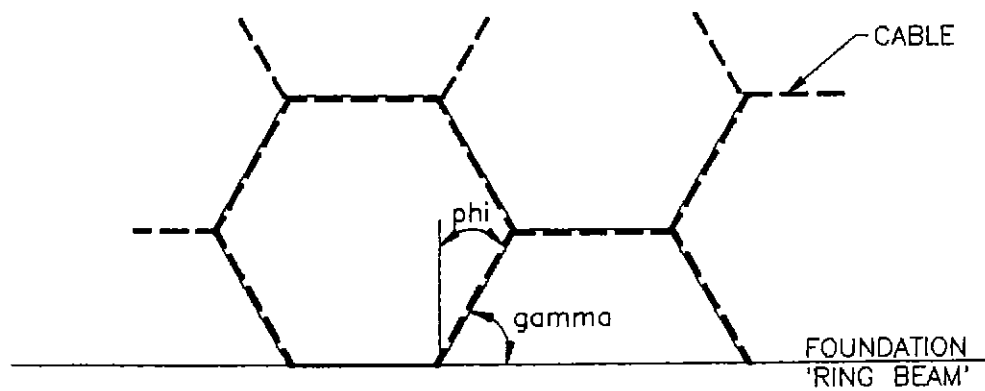
Thus, the angle theta, the complement of alpha which is also shown in Figure 8, is $90 - 63.4 = 26.6$ degrees. Now, looking directly at the North elevation of the dome, another distinct angle would be seen. This angle, phi, is also defined as the angle the cable makes from vertical at the foundation as shown in Figure 8. It is derived from the nature of geodesics and will be discussed hereafter.

The tensile force in the cables, F, at the foundation is then computed by dividing the uplift force, u, associated with a tributary length of the 'ring beam' between two adjacent cables by the product of the cosines of each of the angles, theta and phi. Even though the distance between cables may vary along the 'ring beam', each tributary length of 'ring beam' is approximately equal. In order to determine the remainder of the cable forces throughout the dome, each cable connection must be analyzed.

For the geodesic cable net system, each cable connection consists of the intersection of three cables. Referring to Figure 8, the angles formed by the intersection of the cables are the vertices of the hexagons and pentagons (modules) and vary slightly from point to point due to spherical geometry.



EAST ELEVATION



PARTIAL NORTH ELEVATION

CABLE ORIENTATION

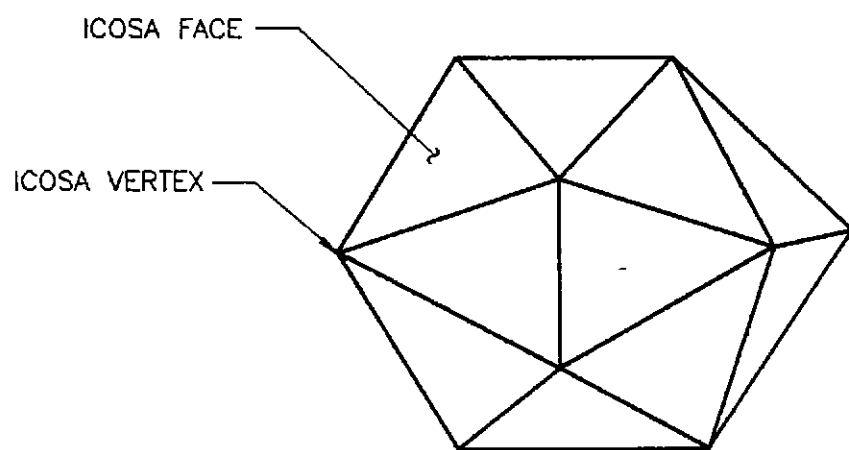
FIGURE 8

The variability of the angles will increase or decrease the forces in some of the cables. Due to the symmetry of the geodesic geometry, the assumption is made that the variation in cable forces is small and will be ignored in the calculations. Since the calculations are based on a minimum breaking strength that is only half of what the ASTM A586 Specification has approved, the small variation of cable forces will not overstress the cables. Therefore, the cable forces tabulated on line 18 of Table 3 are representative of the cable forces at the foundation.

Before actual calculations can be performed, a short review of geodesic geometry is necessary followed by the calculation of cable lengths. Since this introduction is very brief, it is recommended that the reader research this topic with the references given in the bibliography.

GEODESIC GEOMETRY

A geodesic frame is a structural system which distributes loads through a linear arrangement of members placed on the surface of a sphere. The geodesic sphere can be projected from five platonic solids, whose vertices lie in a circumscribed sphere. These platonic solids include the tetrahedron, hexahedron (cube), octahedron, dodecahedron, and the icosahedron. Generally, the icosahedron is the basis for most domes because it best approximates a sphere with its 20 equilateral triangular faces and 12 vertices (see Figure 9).



ICOSAHEDRON

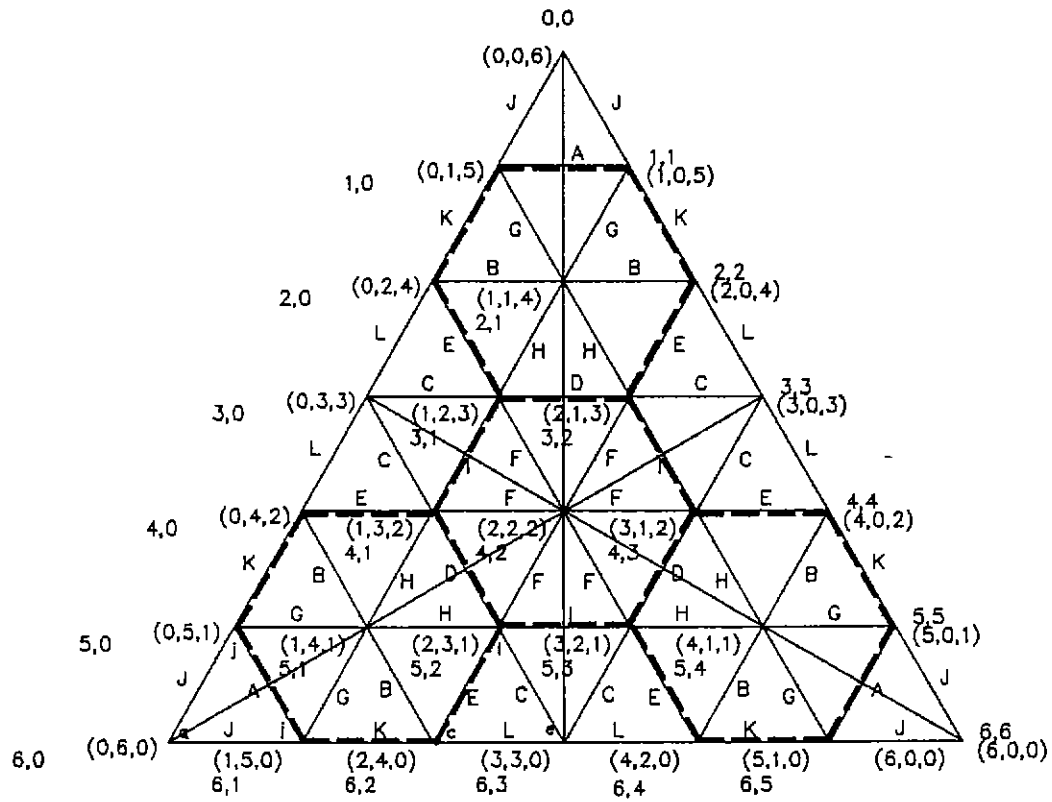
FIGURE 9

Even though the icosahedron best approximates a sphere, its 20 flat faces would be very noticeable on a 500 ft. dome. There have been methods developed for reducing the basic polyhedral form into a larger number of components by subdividing each of the 20 faces of the icosahedron (hereafter called the icosahedron faces) into a three-way grid forming smaller triangles as shown in Figure 10. All of the new vertices formed by this grid are pushed outward until they lie on the surface of the sphere. It is important to maintain a reasonably symmetrical spacing, so that the small triangles created will be nearly equilateral. The number of divisions that are created along the edge of an icosahedron face determines the 'frequency' of the icosahedron. The largest triangle in Figure 10 that bounds all of the other triangular and polygonal shapes is a planar representation of an icosahedron face. Because its edges have been subdivided into 6 parts by grid lines, the figure represents one face of a 6 frequency icosahedron. In addition, 9 and 15 frequency icosahedron faces are shown in Figures 11 and 12, respectively.

There are several methods of generating three way geodesic grids. The one which will be utilized for the geodesic cable net system is commonly referred to as the Class 1 or Alternate breakdown, Method 1. This breakdown takes the icosahedron face and divides its three edges into equal divisions. Each point of subdivision is then connected with a line segment parallel to their respective sides thereby forming a three way grid such that a series of equilateral

a,b = FACE COORDINATES
 (x,y,z) = RECT. COORDINATES
 A = CHORD FACTOR

DASHED LINES REPRESENT CABLES

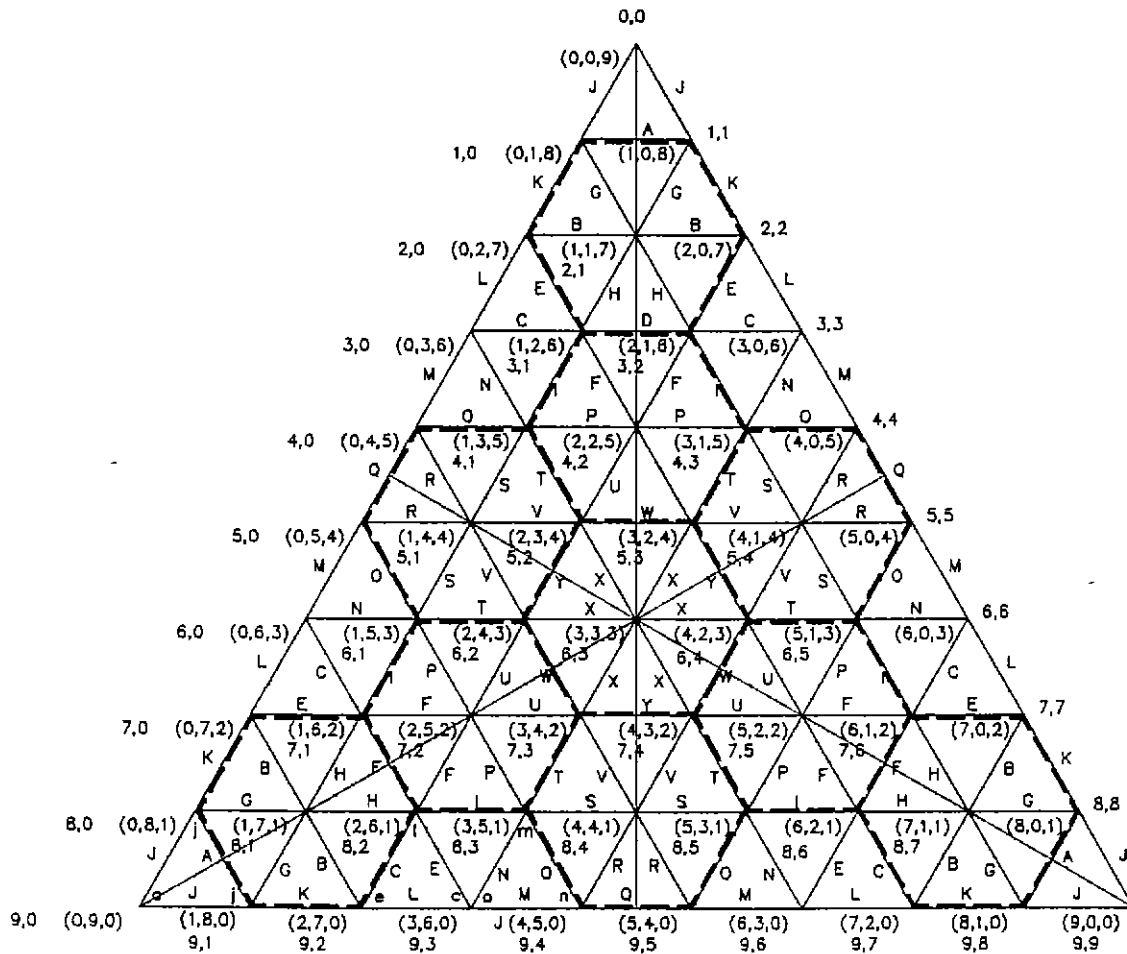


6 - FREQUENCY ICOSAHEDRON FACE

FIGURE 10

a, b = FACE COORDINATES
 (x, y, z) = RECT. COORDINATES
 A = CHORD FACTOR

DASHED LINES REPRESENT CABLES

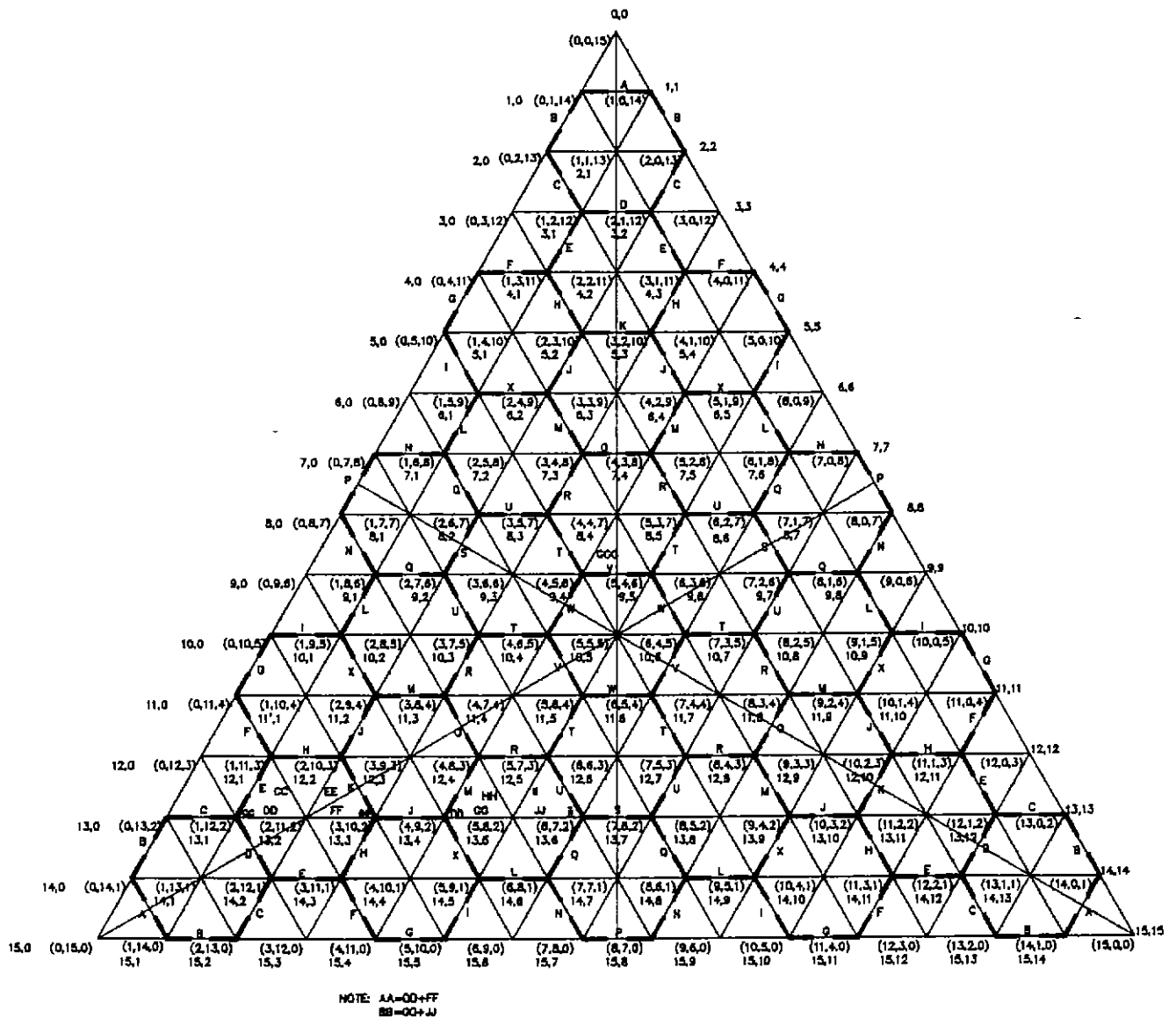


9 - FREQUENCY ICOSAHEDRON FACE

FIGURE 11

a,b = FACE COORDINATES
 (x,y,z) = RECT. COORDINATES
 A = CHORD FACTOR

DASHED LINES REPRESENT CABLES



15 - FREQUENCY ICOSAHEDRON FACE

FIGURE 12

triangles are formed. The grid lines form what are known as 'struts' in geodesics. As shown in Figure 10, the 'struts' that correspond to the cables in the cable net pattern are shown as heavy dashed lines. For a 6 frequency icosahedron, the only 'strut' lengths that must be calculated are 'struts' A, D, E, I, and K where I and D are very similar.

Since Method 1 has been the Class 1 breakdown of choice for most domes in the past, it was used for the geodesic cable net pattern. In "Geodesic Math and How To Use It", by Hugh Kenner, it is stated on pg. 64 that "...by letting the triangle sizes vary more we can keep their shapes more nearly constant." He further states that "the more nearly equilateral a triangle, the more nearly equilibrated should be its resistance to pushes and pulls from various directions." Method 1 would allow the forces in the air form fabric to be transferred more uniformly to the surrounding cables. Utilizing this breakdown, it is possible to obtain the necessary lengths of cable that will be required to form the geodesic cable net. The breakdown provides chord factors which represent 'strut' lengths on a dome having a 1 ft. radius. Therefore, once the chord factor is obtained, the chord length may be calculated by multiplying the chord factor by the radius of the dome in question. Note that the chord factor yields the shortest distance between the end points of the 'strut' (its vertices).

In order to obtain the chord factors, some equations have been developed by using analytical geometry. These equations

require that all of the intersections of the grid lines or 'strut' vertices in an n-frequency icosahedron face have two sets of two-dimensional coordinates as shown in Figure 10. For simplicity, these two sets of coordinates were arbitrarily given names in this thesis. The face coordinates, (a,b), are used for describing the locality of chord factors while the rectangular coordinates, (x,y,z), are used in the following equations for obtaining the actual chord factors. For the equations to yield correct results, both sets of two-dimensional coordinates have to be set up exactly as shown in Figure 10. With the use of the following equations, these coordinates will yield three-dimensional or spherical coordinates which give the location of all the 'strut' vertices on a particular dome. For additional literature on these coordinate systems, please refer to "Geodesic Math and How To Use It", by Hugh Kenner, 1976.

Once the face and rectangular coordinates of the n-frequency icosahedron face are set up, the rectangular coordinates (x,y,z) of each 'strut' vertex are converted by the following formulas, obtained from Kenner, pg. 75:

$$x_1 = x * \sin(72) \quad (4.1)$$

$$y_1 = y + x * \cos(72) \quad (4.2)$$

$$z_1 = f / 2 + z / (2 * \cos(36)) \quad (4.3)$$

where f is the frequency of the icosahedron face.

The spherical icosahedron coordinates are thus calculated by:

$$\phi = \tan^{-1}(x_1 / y_1) \quad (4.4)$$

$$\theta = \tan^{-1}((x_1^2 + y_1^2)^{.5} / z_1) \quad (4.5)$$

where ϕ resembles a meridian of longitude and θ resembles a specification of latitude, both coordinates in degrees. All points whose second coordinate, θ , is 90 degrees, lie on the equatorial great circle. With the exception of these, all points with the same θ lie in the same horizontal plane along which a dome may be truncated such that it sits flat. Note that these coordinates are different than the angles used earlier with the same names. For the 6 frequency icosahedron face, Table 4 shows how the three sets of coordinates correlate with each other and with Figure 10. Tables 5 and 6 correspond to 9 and 15 frequency icosahedron faces, respectively.

Now that the spherical coordinates have been determined, the chord factors for a sphere with a 1 ft. radius may be determined by the following equation, as set forth by Kenner, pg. 60:

$$d = (2 - 2 * (\cos(\theta_1) * \cos(\theta_2) + \cos(\phi_1 - \phi_2) * \sin(\theta_1) * \sin(\theta_2)))^{.5} \quad (4.6)$$

where d is the chord factor and θ_1 is the θ coordinate of the first point, and so forth. The chord factors for the 6 frequency icosahedron face are shown at the bottom of Table 4.

As stated above, the icosahedron has 20 identical faces which have been called icosahedron faces to this point. For a 6 frequency icosahedron, as shown in Figure 10, it was determined that only 5 chord factors must be calculated for determining the cable lengths in the geodesic cable net pattern. Note that the chord Factors A, D, and I each appear

Icosa Frequency =		6													
Face Coordinates		Rectangular Coordinates			Converted Rectangular Coordinates			Spherical Coordinates		Spherical Coordinates					
		x	y	z	x1	y1	z1	phi (rad)	theta (rad)	phi (deg)	theta (deg)				
0,0		0	0	6	0.00000	0.00000	6.70820	0.00000	0.00000	0.000	0.000				
1,0		0	1	5	0.00000	1.00000	6.09017	0.00000	0.16275	0.000	9.325				
1,1		1	0	5	0.95106	0.30902	6.09017	1.25664	0.16275	72.000	9.325				
2,0		0	2	4	0.00000	2.00000	5.47214	0.00000	0.35041	0.000	20.077				
2,1		1	1	4	0.95106	1.30902	5.47214	0.62832	0.28749	36.000	16.472				
3,0		0	3	3	0.00000	3.00000	4.85410	0.00000	0.55357	0.000	31.717				
3,1		1	2	3	0.95106	2.30902	4.85410	0.39071	0.47514	22.386	27.224				
3,2		2	1	3	1.90211	1.61803	4.85410	0.86592	0.47514	49.614	27.224				
4,1		1	3	2	0.95106	3.30902	4.23607	0.27987	0.58248	16.035	39.103				
4,2		2	2	2	1.90211	2.61803	4.23607	0.62832	0.58248	36.000	37.377				
2,2		2	0	4	1.90211	0.61803	5.47214	1.25664	0.35041	72.000	20.077				
3,3		3	0	3	2.85317	0.92705	4.85410	1.25664	0.55357	72.000	31.717				
4,0		0	4	2	0.00000	4.00000	4.23607	0.00000	0.75674	0.000	43.358				
4,3		3	1	2	2.85317	1.92705	4.23607	0.97677	0.68248	55.965	39.103				
4,4		4	0	2	3.80423	1.23607	4.23607	1.25664	0.75674	72.000	43.358				
5,0		0	5	1	0.00000	5.00000	3.61803	0.00000	0.94440	0.000	54.110				
5,1		1	4	1	0.95106	4.30902	3.61803	0.21723	0.88403	12.446	50.651				
5,2		2	3	1	1.90211	3.61803	3.61803	0.48402	0.84626	27.732	48.487				
5,3		3	2	1	2.85317	2.92705	3.61803	0.77262	0.84626	44.268	48.487				
5,4		4	1	1	3.80423	2.23607	3.61803	1.03941	0.84403	59.554	50.651				
5,5		5	0	1	4.75528	1.54508	3.61803	1.25664	0.84440	72.000	54.110				
6,0		0	6	0	0.00000	6.00000	3.00000	0.00000	1.10715	0.000	63.435				
6,1		1	5	0	0.95106	5.30902	3.00000	0.17726	1.06319	10.156	60.916				
6,2		2	4	0	1.90211	4.61803	3.00000	0.39071	1.02988	22.386	59.008				
6,3		3	3	0	2.85317	3.92705	3.00000	0.62832	1.01722	36.000	58.283				
6,4		4	2	0	3.80423	3.23607	3.00000	0.86592	1.02988	49.614	59.008				
6,5		5	1	0	4.75528	2.54508	3.00000	1.07938	1.06319	61.844	60.916				
6,6		6	0	0	5.70634	1.85410	3.00000	1.25664	1.10715	72.000	63.435				
(6,1)		1	5	0	0.95106	5.30902	3.00000	0.17726	1.10715	10.156	63.435				
(6,2)		2	4	0	1.90211	4.61803	3.00000	0.39071	1.10715	22.386	63.435				
(6,3)		3	3	0	2.85317	3.92705	3.00000	0.62832	1.10715	36.000	63.435				
(6,4)		4	2	0	3.80423	3.23607	3.00000	0.86592	1.10715	49.614	63.435				
(6,5)		5	1	0	4.75528	2.54508	3.00000	1.07938	1.10715	61.844	63.435				
Chord Factors		Arc Lengths		Distance between cables along base											
0,0 / 1,0	J	0.162567	0.162747	24.41											
1,0 / 1,1	A	0.190477	0.190766												
1,0 / 2,0	K	0.187383	0.187659	28.15											
1,0 / 2,1	G	0.181908	0.182160												
2,0 / 2,1	B	0.202820	0.203169	30.48											
2,0 / 3,0	L	0.202820	0.203169												
2,0 / 3,1	E	0.198013	0.198338												
2,1 / 3,1	H	0.205908	0.206273												
3,0 / 3,1	C	0.205908	0.206273												
3,1 / 3,2	D	0.215354	0.215772												
3,1 / 4,1	I	0.215354	0.215772												
3,1 / 4,2	F	0.216628	0.217064												
5,5 / (6,5)	A'	0.221668	0.222124												
5,3 / (6,4)	E'	0.271120	0.271958												
6,6 / (6,5)	J'	0.158338	0.158504	23.78											
(6,5 / 6,4)	K'	0.190556	0.190845	28.63											
(6,4 / 6,3)	L'	0.212022	0.212421												
(6,3 / 6,2)	L'	0.212022	0.212421												
(6,4 / 6,2)	2L'	0.421054	0.424228	63.63											
(5,2 / 6,3)	C'	0.205908	0.206273												
(phi)	Face angle f	44.43	degrees												
(phi)	Face angle c'	41.42	degrees												
(phi)	Face angle f'	29.76	degrees												

CHORD FACTORS 6-FREQUENCY ICOSAHEDRON (Class I, Method I)
TABLE 4

Icosa Frequency =		9		Rectangular Coordinates			Converted Rectangular Coordinates			Spherical Coordinates		Spherical Coordinates	
Face Coordinates		x	y	z	x1	y1	z1	phi (rad)	theta (rad)	phi (deg)	theta (deg)		
0,0		0	0	9	0.00000	0.00000	10.06231	0.00000	0.00000	0.000	0.000		
1,0		0	1	8	0.00000	1.00000	9.44427	0.00000	0.10549	0.000	6.044		
1,1		1	0	8	0.95106	0.30902	9.44427	1.25664	0.10549	72.000	6.044		
2,0		0	2	7	0.00000	2.00000	8.82624	0.00000	0.22283	0.000	12.767		
2,1		1	1	7	0.95106	1.30902	8.82624	0.62832	0.18131	36.000	10.388		
3,0		0	3	6	0.00000	3.00000	8.20820	0.00000	0.35041	0.000	20.077		
3,1		1	2	6	0.95106	2.30902	8.20820	0.39071	0.29534	22.386	16.922		
3,2		2	1	6	1.90211	1.61803	8.20820	0.86592	0.29534	49.814	16.922		
4,0		0	4	5	0.00000	4.00000	7.59017	0.00000	0.48501	0.000	27.789		
4,1		1	3	5	0.95106	3.30902	7.59017	0.27987	0.42585	16.035	24.400		
4,2		2	2	5	1.90211	2.61803	7.59017	0.62832	0.40301	36.000	23.091		
5,0		0	5	4	0.00000	5.00000	6.97214	0.00000	0.62214	0.000	35.646		
5,1		1	4	4	0.95106	4.30902	6.97214	0.21723	0.56427	12.446	32.330		
5,2		2	3	4	1.90211	3.61803	6.97214	0.48402	0.53026	27.732	30.382		
5,3		3	2	4	2.85317	2.92705	6.97214	0.77262	0.53026	44.268	30.382		
6,2		2	4	3	1.90211	4.61803	6.35410	0.39071	0.56816	22.386	38.168		
6,3		3	3	3	2.85317	3.92705	6.35410	0.62832	0.55236	36.000	37.377		
8,0		0	8	1	0.00000	8.00000	5.11803	0.00000	1.00168	0.000	57.391		
8,1		1	7	1	0.95106	7.30902	5.11803	0.12939	0.96385	7.414	55.225		
8,2		2	6	1	1.90211	6.61803	5.11803	0.27987	0.93182	16.035	53.378		
8,3		3	5	1	2.85317	5.92705	5.11803	0.44864	0.90958	25.705	52.115		
8,4		4	4	1	3.80423	5.23607	5.11803	0.62832	0.90170	36.000	51.664		
9,0		0	9	0	0.00000	9.00000	4.50000	0.00000	1.10715	0.000	63.435		
9,1		1	8	0	0.95106	8.30902	4.50000	0.11396	1.07716	6.530	61.717		
9,2		2	7	0	1.90211	7.61803	4.50000	0.24468	1.05038	14.019	60.183		
9,3		3	6	0	2.85317	6.92705	4.50000	0.39071	1.02988	22.386	59.008		
9,4		4	5	0	3.80423	6.23607	4.50000	0.54777	1.01867	31.385	58.366		
9,5		5	4	0	4.75528	5.54508	4.50000	0.70887	1.01867	40.615	58.366		
(9,0)		0	9	0	0.00000	9.00000	4.50000	0.00000	1.10715	0.000	63.435		
(9,1)		1	8	0	0.95106	8.30902	4.50000	0.11396	1.10715	6.530	63.435		
(9,2)		2	7	0	1.90211	7.61803	4.50000	0.24468	1.10715	14.019	63.435		
(9,3)		3	6	0	2.85317	6.92705	4.50000	0.39071	1.10715	22.386	63.435		
(9,4)		4	5	0	3.80423	6.23607	4.50000	0.54777	1.10715	31.385	63.435		
(9,5)		5	4	0	4.75528	5.54508	4.50000	0.70887	1.10715	40.615	63.435		
<hr/>													
Chord Factors		Arc Lengths		Distance between cables along base									
1,0/1,1	A	0.123782	0.123882										
1,0/2,0	K	0.117278	0.117343										
2,0/3,1	E	0.122267	0.122344										
3,1/3,2	D	0.137018	0.137126										
3,1/4,1	I	0.135963	0.136068										
4,0/4,1	O	0.136963	0.136968										
4,0/5,0	Q	0.137018	0.137126										
4,1/5,2	T	0.138890	0.140008										
5,2/5,3	W	0.145456	0.145584										
5,2/6,2	Y	0.145455	0.145584										
<hr/>													
8,0/(9,1)	A'	0.144544	0.144671										
8,2/(9,2)	C'	0.177815	0.178050										
8,3/(9,4)	O'	0.214082	0.214503										
(9,0)/(9,1)	J	0.101878	0.101922	25.48									
(9,1)/(9,2)	K'	0.116834	0.116901	29.23									
(9,2)/(9,4)	LM'	0.270050	0.270877	67.72									
(9,4)/(9,5)	Q'	0.143940	0.144065	36.02									
(8,2)/(9,3)	E'	0.198847	0.199176										
(8,3)/(9,3)	N'	0.203158	0.203509										
(phi)	Face angle f	45.19	degrees										
(phi)	Face angle e'	12.40	degrees										
(phi)	Face angle n'	23.22	degrees										

CHORD FACTORS 9-FREQUENCY ICOSAHEDRON (Class I, Method I)
TABLE 5

15-FREQUENCY

15

Face Coordinates	Rectangular Coordinates			Converted Rectangular Coordinates			Spherical Coordinates		Spherical Coordinates	
	x	y	z	x1	y1	z1	phi (rad)	theta (rad)	phi (deg)	theta (deg)
0,0	0	0	15	0.00000	0.00000	16.77051	0.00000	0.00000	0.000	0.000
1,0	0	1	14	0.00000	1.00000	16.15248	0.00000	0.06183	0.000	3.543
1,1	1	0	14	0.95108	0.30902	16.15248	1.25884	0.06183	72.000	3.543
2,0	0	2	13	0.00000	2.00000	15.53444	0.00000	0.12804	0.000	7.336
3,1	1	2	12	0.95108	2.30902	14.91641	0.39071	0.18558	22.368	9.504
3,2	2	1	12	1.90211	1.61803	14.91641	0.86642	0.18558	49.914	9.504
4,0	0	4	11	0.00000	4.00000	14.29837	0.00000	0.27278	0.000	15.829
4,1	1	3	11	0.95108	3.30902	14.29837	0.27987	0.23630	16.035	13.539
5,0	0	5	10	0.00000	5.00000	13.68034	0.00000	0.35041	0.000	20.077
5,2	2	3	10	1.90211	3.61803	13.68034	0.48402	0.29035	27.732	16.838
5,3	3	2	10	2.85317	2.92705	13.68034	0.77282	0.29035	44.268	16.838
6,1	1	5	9	0.95108	5.30902	13.06231	0.17728	0.39158	10.158	22.438
6,2	2	4	9	1.90211	4.61803	13.06231	0.39071	0.36520	22.388	20.925
7,0	0	7	8	0.00000	7.00000	12.44427	0.00000	0.51240	0.000	29.358
7,1	1	6	8	0.95108	6.30902	12.44427	0.14982	0.47378	8.573	27.145
7,3	3	4	8	2.85317	4.92705	12.44427	0.52480	0.42909	30.074	24.585
7,4	4	3	8	3.80423	4.23607	12.44427	0.73174	0.42909	41.926	24.585
8,0	0	8	7	0.00000	8.00000	11.82624	0.00000	0.59475	0.000	34.077
8,2	2	6	7	1.90211	6.61803	11.82624	0.27087	0.52727	16.035	30.211
8,3	3	5	7	2.85317	5.92705	11.82624	0.44864	0.50761	25.705	29.084
9,2	2	7	6	1.90211	7.61803	11.20820	0.24458	0.61110	14.019	35.013
9,4	4	5	6	3.80423	6.23607	11.20820	0.54777	0.57760	31.385	33.094
9,5	5	4	6	4.75528	5.54508	11.20820	0.70887	0.57760	40.615	33.094
10,4	4	6	5	3.80423	7.23607	10.59017	0.48402	0.65740	27.732	37.887
12,1	1	11	3	0.95108	11.30902	9.35410	0.08390	0.88146	4.807	50.504
12,2	2	10	3	1.90211	10.61803	9.35410	0.17728	0.85642	10.158	49.069
12,4	4	8	3	3.80423	9.23607	9.35410	0.39071	0.81820	22.388	48.890
12,5	5	7	3	4.75528	8.54508	9.35410	0.50781	0.80781	29.098	48.273
13,0	0	13	2	0.00000	13.00000	8.73607	0.00000	0.97911	0.000	56.099
13,1	1	12	2	0.95108	12.30902	8.73607	0.07711	0.95497	4.418	54.716
13,3	3	10	2	2.85317	10.62705	8.73607	0.25541	0.91239	14.634	52.276
13,4	4	9	2	3.80423	10.23607	8.73607	0.35583	0.89606	20.388	51.340
13,6	6	7	2	5.70834	8.85410	8.73607	0.57249	0.87841	32.801	50.329
13,7	7	6	2	6.65740	8.16312	8.73607	0.68415	0.87841	39.199	50.329
13,13	13	0	2	12.36373	4.01722	8.73607	1.25884	0.97911	72.000	56.099
(13,1)	1	12	2	0.95108	12.30902	8.73607	0.07711	0.97911	4.418	56.099
(13,2)	2	11	2	1.90211	11.61803	8.73607	0.16228	0.97911	9.298	56.099
(13,3)	3	10	2	2.85317	10.82705	8.73607	0.25541	0.97911	14.634	56.099
(13,4)	4	9	2	3.80423	10.23607	8.73607	0.35583	0.97911	20.388	56.099
(13,5)	5	8	2	4.75528	9.54508	8.73607	0.48220	0.97911	26.482	56.099
(13,6)	6	7	2	5.70834	8.85410	8.73607	0.57249	0.97911	32.801	56.099
(13,7)	7	6	2	6.65740	8.16312	8.73607	0.68415	0.97911	39.199	56.099

Chord Factors Arc Lengths Distance between cables along base

1,0/1,1	A	0.072841	0.072858
1,0/2,0	B	0.089198	0.089211
2,0/3,1	C	0.087980	0.087993
3,1/3,2	D	0.077729	0.077748
3,1/4,1	E	0.073698	0.073715
4,0/4,1	F	0.078985	0.078998
4,0/5,0	G	0.077807	0.077827
4,1/5,2	H	0.075527	0.075548
5,0/6,1	I	0.076188	0.076184
5,2/6,2	J	0.080581	0.080583
5,3/5,3	K	0.082335	0.082356
6,1/7,1	L	0.082982	0.082989
6,2/7,2	M	0.082170	0.082193
7,0/7,1	N	0.080581	0.080583
7,3/7,4	O	0.085903	0.085929
7,0/8,0	P	0.082335	0.082356
7,1/8,2	Q	0.082170	0.082193
7,3/8,3	R	0.085903	0.085929
8,2/9,2	S	0.085903	0.085929
8,3/9,4	T	0.086815	0.086842
8,2/8,3	U	0.085907	0.085983
9,4/9,5	V	0.087870	0.087896
9,4/10,4	W	0.087870	0.087896
6,1/6,2	X	0.082982	0.082989

12,1/(13,1)	E'	0.087783	0.087802
12,2/(13,3)	K'	0.137331	0.137439
12,4/(13,4)	M'	0.163008	0.163189
12,5/(13,6)	U'	0.178459	0.178697
13,0/(13,1)	C'	0.083987	0.083988
(13,1)/(13,3)	AA'	0.147790	0.147925
(13,3)/(13,4)	J'	0.083315	0.083329
(13,4)/(13,6)	BB'	0.179475	0.179717
(13,6)/(13,7)	S'	0.092829	0.092962
(12,1)/(13,2)	CC'	0.118021	0.118066
(13,1)/(13,2)	DD'	0.070899	0.070884
(12,2)/(13,2)	EE'	0.123178	0.123256
(13,2)/(13,3)	FF'	0.077288	0.077287
(13,4)/(13,5)	GG'	0.088246	0.088274
12,4/(13,5)	HH'	0.170086	0.170292
12,5/(13,5)	II'	0.174893	0.175117
(13,5)/(13,6)	JJ'	0.091483	0.091525

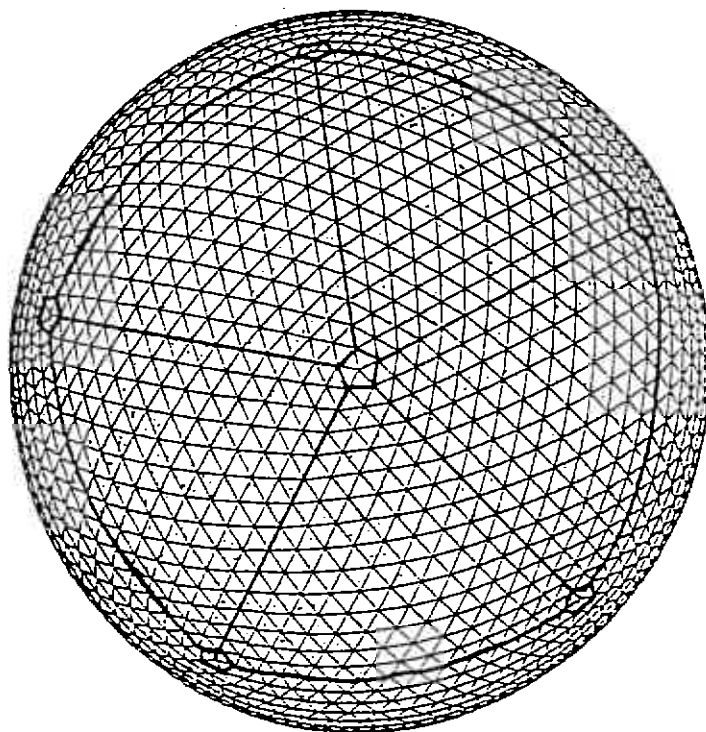
(phi)	Face angle cc'	4.53	degrees
(phi)	Face angle ee'	27.07	degrees
(phi)	Face angle hh'	10.58	degrees
(phi)	Face angle ii'	17.15	degrees

CHORD FACTORS 15-FREQUENCY ICOSAHEDRON (Class I)
TABLE 6

3 times in the icosahedron face and the chord factors E and K each occur 6 times in each icosahedron face. The set of 5 chord factors is repeated 20 times on an icosahedron which amounts to one set for each face. However, for a dome, only a few of the icosahedron faces are actually utilized.

The number of icosahedron faces which are utilized depends on the angle alpha which is shown in the East Elevation of Figure 8. If a hemispherically shaped dome is built using a geodesic cable net, then a combination of 5 complete or whole icosahedron faces and 10 partial or truncated icosahedron faces will be used to describe its shape. The geometry associated with this statement will be shown later. The 5 whole icosahedron faces which symmetrically surround the apex of the dome will hereafter be referred to as the icoscap as shown in Figure 13 for a 16 frequency icosahedron, courtesy of "Domebook 2", by Lloyd Kahn and others. Note that the cables in the geodesic cable net will form pentagon shapes at the 30 icosahedron vertices and will form hexagons elsewhere. A common soccer ball could be classified as a 3 frequency icosahedron, but its 'struts' follow the surface of a sphere instead of being linear.

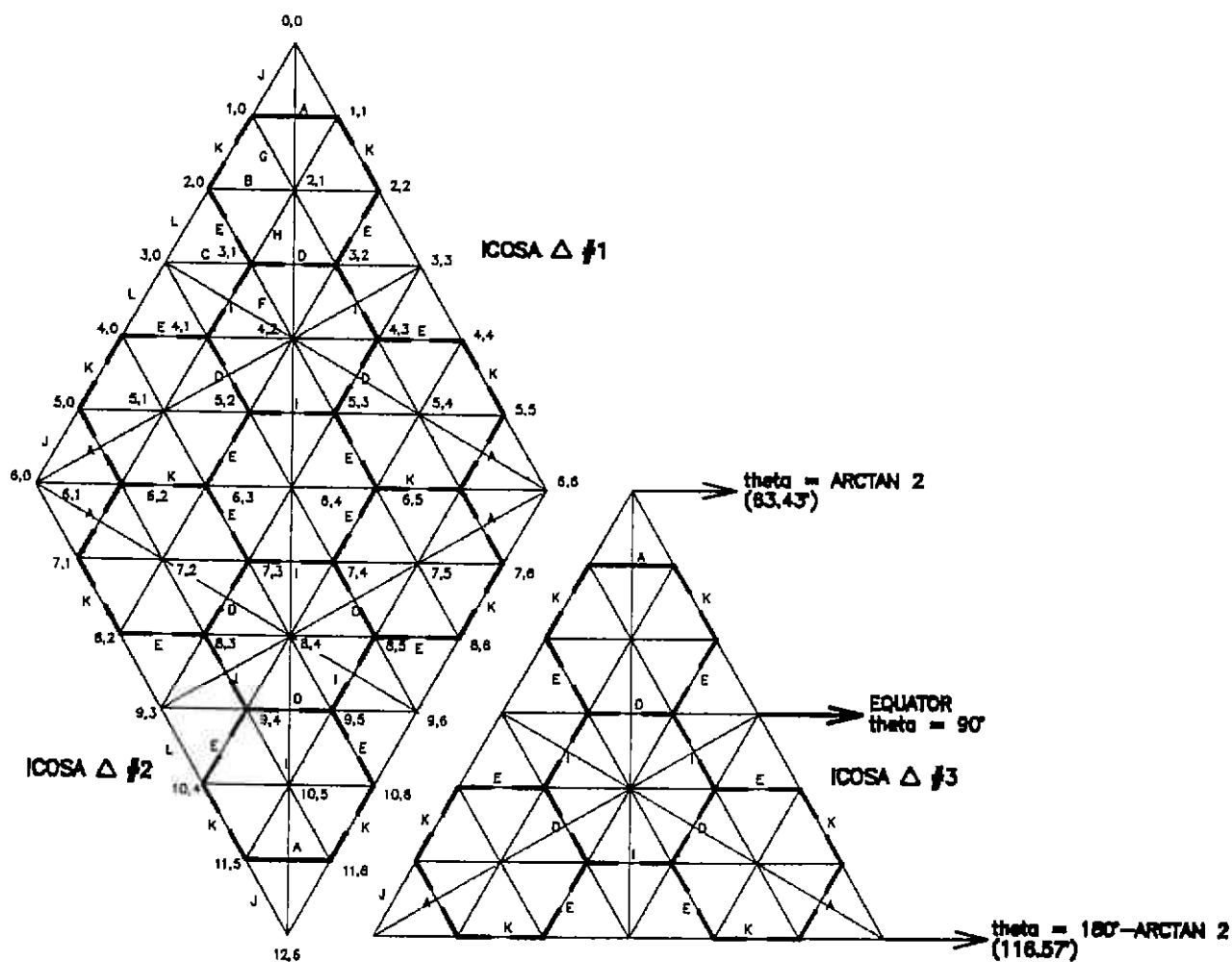
Below each of the 5 icosahedron faces in the icoscap, there is an inverted icosahedron face which contains the same chord factors as the icosahedron faces within the icoscap. Adjacent to the inverted icosahedron face, an upright icosahedron face is situated at the same latitude on both sides as shown in Figure 14 for a 6 frequency icosahedron. The icosahedron triangle #1 represents one of the icosahedron faces in the icoscap. The icosahedron triangle #2



GEODESIC CABLE PATTERN ICOSCAP

FIGURE 13

a, b = FACE COORDINATES
 (x, y, z) = RECT. COORDINATES
 A = CHORD FACTOR



COMBINATION OF ICOSAHEDRON FACES

FIGURE 14

represents the inverted icosahedron face while icosahedron triangle #3 represents the icosahedron face adjacent to icosahedron triangle #2. The arrows at the right edge of the icosahedron triangles in Figure 14 represent truncation planes where the icosahedron may be cut such that the dome will sit flat on a foundation.

The easiest truncation plane to use is the equatorial line at $\theta = 90$ degrees because all of the grid line vertices along that line have a θ coordinate of 90 degrees, meaning that all the vertices lie in the same horizontal plane. The vertices along any other truncation plane do not contain matching θ coordinates. For example, in Table 4, the θ coordinates for the 'arctan 2' truncation plane which includes grid line vertices 6,0 through 6,6 (see Figure 14) vary from 63.4 degrees to 58.3 degrees. As shown later, the solution to a flat foundation is to alter the geodesic geometry by forcing the θ coordinate of such vertices to correspond with each other. Generally, the outer vertices (6,0 and 6,6) are held constant and the interior vertices are adjusted in order to lessen the mathematics involved. Notice that the truncation plane that corresponds to the θ coordinate of 63.4 degrees will always occur at the base of the icosahedron triangle #1, i.e. at the base of the icosahedron.

With this introduction to geodesic geometry, cable lengths will be discussed next followed by calculations of cable forces.

CABLE LENGTHS

Similar to the radial cable net system, the individual length of each cable is an essential component of the entire system. The cable lengths must be exact such that they are stiff and fit firm to the air form. Using arc length principles, these lengths may be calculated precisely by using the central angle of a 'strut'. The central angle is formed by two radii of the icosahedron passing through the end points of the 'strut' and may be measured at the radius or central point of the icosahedron.

It is now convenient to again consider the 500 ft. dome such that comparisons between each cable net system may be made. As shown on line 4 of Table 3, a 9 frequency icosahedron was selected for the geometry of the 500 ft. dome geodesic cable net. The frequency was selected such that the cable lengths were approximately 30 to 35 ft. As mentioned above in the section on radius of curvature of this chapter, the 30 to 35 ft. cable lengths comply with the recent recommendations made by an air form manufacturer that the hexagons and pentagons be sized such that they are circumscribed by a circle with an approximate diameter of 60 to 70 ft. in order to allow sufficient area for the modules to 'balloon out'. In order for the cables to align correctly between adjacent icosahedron faces, only the icosahedron frequencies which are multiples of three will work as geodesic cable nets.

As shown in Figure 11 by the heavy dashed lines, the chord factors which must be calculated for the 9 frequency

icosahedron are labeled as A, D, I, E, K, O, Q, T, W and Y. Some of these chord factors are equal to each other as shown at the bottom of Table 5. For simplification, only the chord factor A near the top of the icosahedron face will be utilized for sample calculations. As shown in Figure 11, the chord factor A appears near each vertex of the icosahedron face and forms one side of the pentagon shape that occurs at such vertices. If the left end of chord factor A is arbitrarily designated as the first end point, then the face coordinates and the rectangular coordinates of the first point are (1,0) and (0,1,8), respectively (see Figure 11). Likewise, the face coordinates and rectangular coordinates of the second point are (1,1) and (1,0,8), respectively. Using Equations (4.1), (4.2), and (4.3), the converted rectangular coordinates for the first point, as shown in Table 5, are:

$$\begin{aligned}
 x_1 &= x * \sin(72) \\
 &= 0 * \sin(72) \\
 &= 0 \\
 y_1 &= y + x * \cos(72) \\
 &= 1 + 0 * \cos(72) \\
 &= 1 \\
 z_1 &= f / 2 + z / (2 * \cos(36)) \\
 &\quad 9 / 2 + 8 / (2 * \cos(36)) \\
 &\quad 9.44
 \end{aligned}$$

Likewise, the converted rectangular coordinates for the second point are $(x_2, y_2, z_2) = (0.951, 0.309, 9.44)$ as shown in Table 5.

The spherical icos coordinates for the first point may now be calculated by using Equations (4.4) and (4.5):

$$\begin{aligned}
 \phi_{i_1} &= \text{TAN}^{-1}(x_1 / y_1) \\
 &= \text{TAN}^{-1}(0 / 1) \\
 &= 0 \text{ degrees} \\
 \theta_{i_1} &= \text{TAN}^{-1}((x_1^2 + y_1^2)^{.5} / z_1) \\
 &= \text{TAN}^{-1}((0^2 + 1^2)^{.5} / 9.44) \\
 &= 6.04 \text{ degrees}
 \end{aligned}$$

Likewise, the spherical coordinates, in degrees, for the second point are $(\phi_{i_2}, \theta_{i_2}) = (72, 6.04)$ as shown in Table 5.

With the spherical coordinates of each end point of chord factor A and using Equation (4.6), the actual chord factor A, as shown at the bottom of Table 5, is.

$$\begin{aligned}
 d &= (2 - 2 * (\text{COS}(\theta_{i_1}) * \text{COS}(\theta_{i_2}) + \text{COS}(\phi_{i_1} - \phi_{i_2}) * \text{SIN}(\theta_{i_1}) * \text{SIN}(\theta_{i_2})))^{.5} \\
 &= (2 - 2 * (\text{COS}(6.04) * \text{COS}(6.04) + \text{COS}(0 - 72) * \text{SIN}(6.04) * \text{SIN}(6.04)))^{.5} \\
 &= 0.124
 \end{aligned}$$

Since this chord factor represents the 'strut' length for a dome or a sphere with a 1 ft. radius, the actual 'strut' length A is $250 * 0.124 = 31$ ft. Since the cable is designed to follow the radius of curvature of the air form fabric, the 'strut' length must be converted to an arc length by using the central angle. By trigonometry, the central angle, delta, is calculated by the following equation:

$$\text{delta} = 2 * \text{SIN}^{-1}(d/2) \quad (4.7)$$

where d is the chord factor.

If chord factor A is equal to 0.124, then the angle δ from Equation (4.7) is:

$$\begin{aligned}\delta &= 2 * \sin^{-1}(0.124/2) \\ &= 7.11 \text{ degrees} \\ &= 0.124 \text{ radians}\end{aligned}$$

Now, by arc length principles, the arc length associated with chord factor A is $0.124 * 250 = 31$ ft. as shown on line 5 of Table 7. Notice that the difference between the 'strut' length and the arc length is insignificant due to the large radius of curvature of the dome. The remainder of the arc lengths associated with each chord factor in a 9 frequency 500 ft. dome are also tabulated on line 5 of Table 7.

Referring to line 6 of Table 3, the angle α (see Figure 8) for this dome is 63.4 degrees which represents both the coordinate θ and the truncation plane. Since the selected truncation plane occurs at 63.4 degrees, only the 5 icosahedron faces within the icoscap will be included in the dome geometry. Note in Figure 11 that the truncation plane involves grid line vertices 9,0 through 9,9. Since the θ coordinates of these grid line vertices are not all equal, the θ coordinate for vertices 9,1 through 9,8 will be adjusted to that of vertices 9,0 and 9,9 which is 63.4 degrees. Referring to Figure 11, the cable lengths which will be altered correspond to the chord factors A , C , and O . These adjusted chord factors are designated as A' , C' , and O' and are also tabulated at the bottom of Table 5. Adjusted chord

1	Diameter at base of Dome (ft.) =	300	300	400	500	800	800
2	Number of cables tied to foundation =	20	20	30	30	40	40
3	Dome height, H (ft.) =	82.71	150.00	123.61	164.51	156.85	213.13
4	Icosaheron frequency utilized, f =	6	6	9	9	15	15
5	Geodesic cable lengths, arc distance (ft.):						
	Cable A =	28.61	28.61	24.78	30.97	21.80	29.08
	Cable B =					19.88	26.48
	Cable C =					20.37	27.16
	Cable D =	32.37	32.37	23.46	34.28	23.32	31.10
	Cable E =	32.37	32.37	24.45	34.02	22.86	30.47
	Cable F =	29.75	29.75	27.40	30.59	22.11	29.49
	Cable G =					23.70	31.60
	Cable H =					23.29	31.05
	Cable I =					22.86	30.22
	Cable J =					24.17	32.23
	Cable K =	28.15	28.15	27.19	29.34	24.71	32.94
	Cable L =					24.90	33.19
	Cable M =					24.66	32.88
	Cable N =					24.17	32.23
	Cable O =			27.19	34.02	25.78	34.37
	Cable P =					24.71	32.94
	Cable Q =			27.40	34.28	24.66	32.88
	Cable R =					25.70	34.27
	Cable S =					25.78	34.37
	Cable T =			27.98	35.00	25.99	34.66
	Cable U =					25.70	34.27
	Cable V =					26.37	35.16
	Cable W =			29.09	36.40	26.37	35.16
	Cable X =					24.80	33.19
	Cable Y =			29.09	36.40		
6	Num. of icos triangles in icoscap:	5	5	5	5	5	5
7	Num. of whole cables in ea. tri. in icoscap:						
	Cable A =	1	3	1	1	1	1
	Cable B =					2	2
	Cable C =					2	2
	Cable D =	3	3	3	3	1	1
	Cable E =	3	3	6	6	4	4
	Cable F =	4	6	4	4	2	2
	Cable G =					4	4
	Cable H =					4	4
	Cable I =					4	4
	Cable J =					4	4
	Cable K =	4	6	4	4	1	1
	Cable L =					4	4
	Cable M =					4	4
	Cable N =					4	4
	Cable O =			4	4	3	3
	Cable P =					2	2
	Cable Q =			2	2	4	4
	Cable R =					6	6
	Cable S =					2	2
	Cable T =			6	6	6	6
	Cable U =					4	4
	Cable V =					3	3
	Cable W =			3	3	3	3
	Cable X =						
	Cable Y =			3	3		
8	Qty. of #2 trunc. icos triangles in cable net:	0	5	0	0		
9	Num. of whole cables per #2 trunc. icos tri.:						
	Cable A =	0	2	0	0		
	Cable D =	0	2	0	0		
	Cable I =	0	3	0	0		
	Cable E =	0	4	0	0		
	Cable K =	0	4	0	0		
10	Qty. of #3 trunc. icos triangles in cable net:	0	5	0	0		
11	Num. of whole cables per #3 trunc. icos tri.:						
	Cable A =	0	1	0	0		
	Cable D =	0	0	0	0		
	Cable I =	0	0	0	0		
	Cable E =	0	2	0	0		
	Cable K =	0	2	0	0		

TOTAL CABLE LENGTHS FOR VARIOUS DOME SIZES

TABLE 7

(Continued next page)

	Diameter at base of Dome (ft.) =	300	300	400	500	600	800
	Number of cables tied to foundation =	20	20	30	30	40	40
	Dome height, H (ft.) =	92.71	150.00	123.81	154.51	159.85	213.13
	Icosaheran frequency utilized, f =	6	6	9	9	12	15
12	Total number of whole cables in cable net:						
	Cable A =	5	15	5	5	5	5
	Cable B =					10	10
	Cable C =					10	10
	Cable D =	15	15	15	15	5	5
	Cable I =	15	15	30	30	20	20
	Cable E =	20	30	20	20	10	10
	Cable F =					20	20
	Cable G =					20	20
	Cable H =					20	20
	Cable J =					20	20
	Cable K =	10	20	20	20	5	5
	Cable L =					20	20
	Cable M =					20	20
	Cable N =					20	20
	Cable O =			20	20	15	15
	Cable P =					10	10
	Cable Q =			10	10	20	20
	Cable R =					30	30
	Cable S =					10	10
	Cable T =			30	30	30	30
	Cable U =					20	20
	Cable V =					15	15
	Cable W =			15	15	15	15
	Cable X =						
	Cable Y =			15	15		
13	Adjusted cable lengths, arc distance (ft.):						
	Cable A' =	33.32		28.91	36.17		
	Cable C' =				44.51		
	Cable E' =	40.79		35.56		29.34	39.12
	Cable K' =					41.23	54.98
	Cable M' =					48.98	65.28
	Cable O' =			42.82	53.63		
	Cable U' =					53.61	71.48
14	Num. of adjusted cables in icoscap:						
	Cable A' =	10		10	10		
	Cable C' =				10		
	Cable D' =						
	Cable I' =						
	Cable E' =	10		10		10	10
	Cable K' =					10	10
	Cable M' =					10	10
	Cable O' =			10	10		
	Cable U' =					10	10
	Total cable length, arc distance (ft.) =	2731.87	2855.89	5904.00	7396.12	8723.38	7300.51

TOTAL CABLE LENGTHS FOR VARIOUS DOME SIZES

TABLE 7

factors for chord factors J, K, L, M, and Q are also tabulated at the bottom of Table 5. They represent various distances along the foundation of the dome between the cables.

The number of whole cables that correspond to each chord factor in a 9 frequency icosahedron are tabulated on line 7 of Table 7. Whole cables signify those that are not adjusted. Note that on line 7, there is only 1 whole cable associated with chord factor A per icosahedron face which comes out to 5 whole cables in the icosahedron. The other two cables associated with chord factor A in each icosahedron face have been adjusted to the length specified on line 13 of Table 7. Notice that cable A' is over 6 ft. longer than cable A. As shown on line 14, there are a total of 10 adjusted cables associated with chord factor A within the icosahedron. Therefore, the total length of cable A and cable A' combined is $31.0 * 5 + 36.2 * 10 = 517$ ft. Note that this length does not account for cable connections and elastic elongation. Once the connectors are sized, the necessary reductions in cable lengths should be made accordingly. The lengths of the remainder of the cables in the cable net are calculated similarly. Note that the total length of cable for a 9 frequency 500 ft. dome is just under 7400 ft. as shown on line 21 of Table 3. The reader may verify this length using Table 7. At \$5.00 per ft. of installed cable, this length of cable with its required connections would cost approximately \$40,000, as shown on line 24 of Table 3. Notice that the geodesic cable net provides a

very efficient use of cables for the 600 and 800 ft. domes in comparison to the radial cable net for the same domes.

At the beginning of this chapter, the height of the 500 ft. dome was given along with the radius of curvature of the cable net. Following are some calculations that verify the values previously given. As mentioned earlier, the truncation plane for the 500 ft. dome was selected to be 63.4 degrees based on the geodesic configuration of the cable net. The selected truncation plane dictates the height of the finished dome. Since the theta coordinate of the truncation plane corresponds to the angle alpha shown in Figure 8, the radius of curvature may be calculated by dividing the horizontal dome radius, r , by the sine of the angle alpha. Therefore, the radius of curvature of the cable net for the 500 ft. dome, as tabulated on line 5 of Table 3, is approximately:

$$\begin{aligned} R &= 250 / \sin(63.4) \\ &= 280 \text{ ft.} \end{aligned}$$

Equation (3.1) is then used to solve for the height of the dome, H , where R is used in lieu of r . The result gives a dome height of 155 ft. as tabulated on line 3 of Table 3.

CALCULATION OF CABLE FORCES

Now that a review of geodesic geometry has transpired, it is possible to calculate the cable forces for the 500 ft. dome. As shown in Figure 11, there are 6 cables in each icosahedron face that connect to the domes foundation at the 63.4 degree truncation plane. With 5 icosahedron faces in the icoscap, the

total number of cables that connect to the 'ring beam' is 30, as shown on line 2 of Table 3. In the section on cable forces in this chapter, the total uplift force on the dome, U , and the angle θ , as shown in Figure 8, were calculated as 1790 kips and 26.6 degrees, respectively. The second angle, ϕ , also shown in Figure 8, is derived from the nature of geodesic geometry. The angle ϕ , more commonly known as a 'face angle' in geodesics, represents the angle formed by the grid lines or 'struts' that divide the icosahedron face into particular frequencies. These angles may be determined by the law of cosines where the sides of the triangles (formed by the grid lines) are simply the chord factors that bound the faces. The reader may refer to Chapter 19 of "Geodesic Math and How to Use It", by Hugh Kenner for additional information on face angles.

The angle ϕ will vary from cable to cable, but only that angle associated with the adjusted chord factor A will be calculated here. In fact, the various ϕ angles for the adjusted cables at the foundation are tabulated at the bottom of Tables 4, 5, and 6 for 6, 9, and 15 frequency icosahedrons, respectively. As shown in Figure 11, the angle ϕ for chord factor A is the complement of the angle, j , that is described by the face coordinates 8,0, 9,1, and 9,0. The angle, j , corresponds to the angle γ shown in Figure 8. The chord factors bounding this triangular face are A' , J' , and J' and the corresponding face angles are a' , j' , and j' . Since these are adjusted chord factors, the adjusted angle j' must be

calculated. Using the law of cosines, this angle, as shown on Table 7, is:

$$\begin{aligned} j' &= \cos^{-1} \left((J'^2 + A'^2 - J'^2) / (2 * J' * A') \right) & (4.8) \\ &= \cos^{-1} \left(((0.1445)^2) / (2 * 0.1019 * 0.1445) \right) \\ &= 44.8 \text{ degrees.} \end{aligned}$$

Since the angle phi is the complement of the angle gamma, phi is $90 - 44.8 = 45.2$ degrees. Note that this angle is quite extreme compared to the other phi angles at the base of the 9 frequency icosahedron face as shown at the bottom of Table 5.

The cable force, F, as shown on line 17 of Table 3, is therefore:

$$\begin{aligned} F &= U / (NC * \cos(\alpha) * \cos(\phi)) & (4.9) \\ &= 1790 / (30 * \cos(26.6) * \cos(45.2)) \\ &= 95 \text{ kips} \end{aligned}$$

where

U = Total uplift force on air form, kips.

NC = Number of cables required at foundation.

Just before the review of geodesic geometry, it was stated that the variability of the face angles throughout the dome will increase or decrease the forces in some of the cables. To account for this, a detailed study of each cable joint should be conducted to obtain the necessary angles for cable force calculations. The cable forces for the rest of the domes shown on line 17 of Table 3 are based on the maximum angle of phi as tabulated at the bottom of Tables 4, 5, and 6 for 6, 9, and 15 frequency icosahedrons, respectively.

An additional method may be used to approximate the forces in the cables for the geodesic cable net. This method involves a direct calculation of the cable forces for any cable throughout the cable net by using the same principle that Equation (2.1) is based upon. To utilize the equation correctly, substitute the cable force, F , in lieu of N , and replace the air form radius of curvature, r_a , with the cable radius of curvature, R , which was calculated as 279.5 ft. In addition, the air pressure, p , is modified such that it accounts for a tributary width of module upon which the air pressure is being applied. The tributary width of module is basically a one foot wide strip which continues perpendicularly from the cable to a point midway between the cable and the apex of each adjacent module. This area represents the amount of load that the cable experiences within one foot of length.

As stated in the section on air form surface areas of this chapter, the surface area of each module in the 500 ft. dome may be approximated by modeling a dome with a circular base diameter of 72.8 ft. This diameter represents a circle that is circumscribed about the center most hexagon (module) in the 9 frequency icosahedron, which represents the largest module on the dome. As shown in Figure 11, the module is bounded by chord factors W and Y . The corresponding chord lengths on a 500 ft. dome are both 36.4 ft. as shown on line 5 of Table 7.

Previously, the surface area of this module was determined to be approximately 4300 ft². The tributary loaded area per module associated with each cable consists of a triangle bounded by grid lines within the hexagon shape. Since the grid lines form 6 triangles or faces within the hexagon shaped module, each cable has a tributary width of air form equal to $4300 / 6 = 720$ ft² within each module. When this surface area is divided by the length of the cable, then the result represents the tributary width of loaded air form per module measured perpendicular to the cable. The tributary width associated with each module is thus $720 / 36.4 = 19.8$ ft. Since each cable is bounded by two modules, the tributary width of air form associated with each cable is $19.8 * 2 = 39.6$ ft.

By implementing Equation (2.1) with the aforementioned substitutions, The force in cable W on the 500 ft. dome is therefore:

$$\begin{aligned} F &= (9.1 * 39.6 * 279.5) / 2 \\ &= 50.4 \text{ kips} \end{aligned}$$

This force may be compared to the uplift force, u, associated with a tributary length of 'ring beam' between cables at the foundation. As shown in Figure 6, the cable force, F, is equal to u for hemispherical domes. For the 500 ft. dome, the uplift force, u, was calculated at 59.5 kips as shown on line 8 of Table 3. Note that the tributary width method which yielded the 50.4 kip cable force seems to produce lower values. It is a more approximate method since the air

form surface areas were estimated. Since the cable forces are higher using the uplift method, they are used in all of the spreadsheet calculations. It is recommended that a small dome, say 50 ft. in diameter, be constructed using cable nets such that the cable forces may be measured and compared with those given in Table 3.

STABILITY

The stability of the air-supported form for the geodesic cable net system is much superior than that of the radial cable net system due to the equivalent radius of curvature of each of the cables in the geodesic pattern. This equivalence in cable radius of curvature is derived from the similarity in size and shape of the modules. The more similar the modules, the more uniformly distributed is the internal pressure amongst the cables. With this similarity in shape and size, it may be possible to isolate one of these shapes (modules) and analyze it as a 'smaller' dome with a polygonal 'ring' beam. In contrast, the radial cable net system did not provide a uniform distribution of internal pressure to the cables, thus inducing a very large radius of curvature at the apex of the air form as discussed at the end of Chapter 3. However, other stability concerns induced by insufficient amounts of or variations of air pressure within the air form or non-uniform concrete application still remain significant for the geodesic cable net system.

As with the radial cable net system, stress concentrations will exist in the concrete below the geodesic cables where the air form experiences an abrupt change of radius of curvature. These 'joints' or valley's must be thickened such that they act as 'ring beams' for each of the modules formed by the 'ballooning out' of the air form within the pentagon and hexagon patterns and as 'ribs' for the complete geodesic dome. Again, the sizes of these 'ribs' will have to be determined by a finite element analysis as mentioned in Chapter 1.

As concluded in Chapter 3, the compression in the 'ribs' as well as the cable weights on the air form are small. After the 'ribs' are in place, the modules can be constructed. If the shotcrete is applied uniformly in 1/2 in. maximum layers, and if the air form that 'balloons out' maintains a minimum ratio of rise to span of 1:4, and the air pressure is sufficient and continuously monitored, then the stability of the dome during construction should be adequate. The long term stability of the dome is left for the actual structural design.

CABLE CONNECTIONS

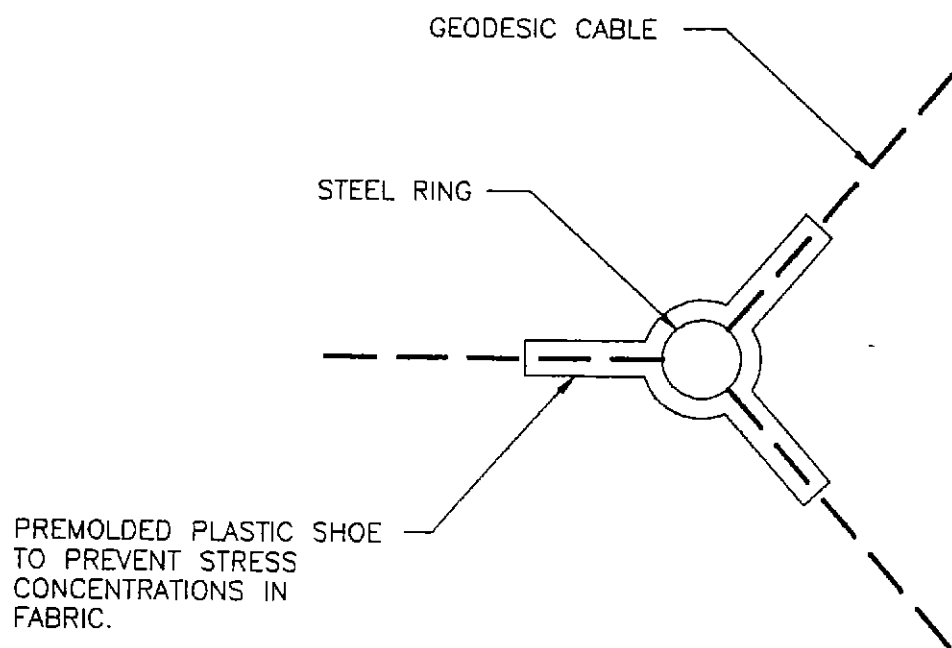
The connection of the cables to each other and to the foundation is left to the air form manufacturer. However, a few recommendations on these connections are made. For cable to cable connections, a three-way pre-manufactured splice chuck that is field attached could be a feasible solution.

However, the angles formed by the intersection of the cables may vary from connection to connection, making it difficult to use a typical splice chuck with different cable configurations.

Another alternative for cable to cable connections is the use of circular steel rings at the cable junctions. Pre-manufactured clevises or turnbuckles may be used to connect the cables to the steel rings. To complete the connection, the cables would have to be threaded at the each end. At the foundation, eye hooks may be embedded into the concrete 'ring beam' to a depth sufficient to resist the uplift forces. Again, a clevis may be used to connect the cable to the eye hook.

In addition to the cable connections, it is imperative that the cables be connected to the air form at the cable junctions as well as at particular increments along the length of each cable. At the cable junctions, a premolded plastic shoe may be attached to the air form, as shown in Figure 17, such that stress concentrations in the fabric are prevented. Between the cable junctions, additional plastic shoes or 'keepers' may be attached to the air form so that the cables maintain their relative position with respect to the air form.

In conclusion, the actual connections are left to the air form manufacturer even though some suggestions have been made. It is imperative, however, that the connections are capable of resisting the tensile forces in the cables.



TYPICAL CABLE CONNECTION

FIGURE 15

CHAPTER 5

CONCLUSIONS

Recently, there has been an interest to build large concrete domes in excess of 300 ft. in diameter. Unfortunately, the air-supported forming system has placed an upper limit on the size of a thin shell concrete dome that can be safely constructed. By shortening the air form's radius of curvature with the use of cable nets, this thesis has provided a solution whereby air-supported forms may be used for such large concrete domes. Even though some of the research proved that the radial cable net system would not provide a valid solution to the proposed problem, it was presented so that its limitations were not overlooked by an interested air-form manufacturer or contractor.

Many of the solutions presented in this thesis are theoretical. Since the air form fabric behaves differently from one size of dome to another, small scale models or prototypes of the proposed style of domes were not considered. Hence, there will likely be a certain hesitancy in applying the solutions that are proposed in this thesis. For this reason, it is highly recommended that a 50 ft. diameter dome be constructed first such that comparisons may be made between the cable forces presented here and those on an actual dome. A 300 ft. diameter dome should then be built in order to measure the air form stresses within the modules. Other values that should be verified include cable lengths, cable

forces, radius of curvature values as well as the overall cost of the project.

The overall cost of a dome project using air-supported forms is very efficient. Recently, an air form manufacturer suggested that the total cost of building a 500 ft. dome such as the one analyzed in Chapter 4, would be about \$10 million. This may seem like a lot of money, but when comparing it to the cost of some of the traditionally formed domes of the past, the cost of building a dome using air-supported forms and cable nets is very reasonable. For example, the Kingdome, a 650 ft. diameter thin shell concrete dome built in Seattle, Washington during the mid 1970's, cost a total of \$58.2 million according to reference 7 on page 344 of "Thin Shell Concrete Structures", by David Billington. Neglecting the time value of money, the proposed 500 ft. dome would only cost a sixth of what it cost to build the Kingdome. The economy of the air-supported forming techniques is so superior that it has warranted the need to build large domes using air-supported form techniques.

In conclusion, it is recommended that a finite element analysis be developed soon such that the structural behavior of the domes presented in this thesis may be determined. The finite element analysis should give more precise results on such items as stability, cable forces, and air form surface areas. The structural analysis will be imperative in confirming the conclusions that have been drawn.